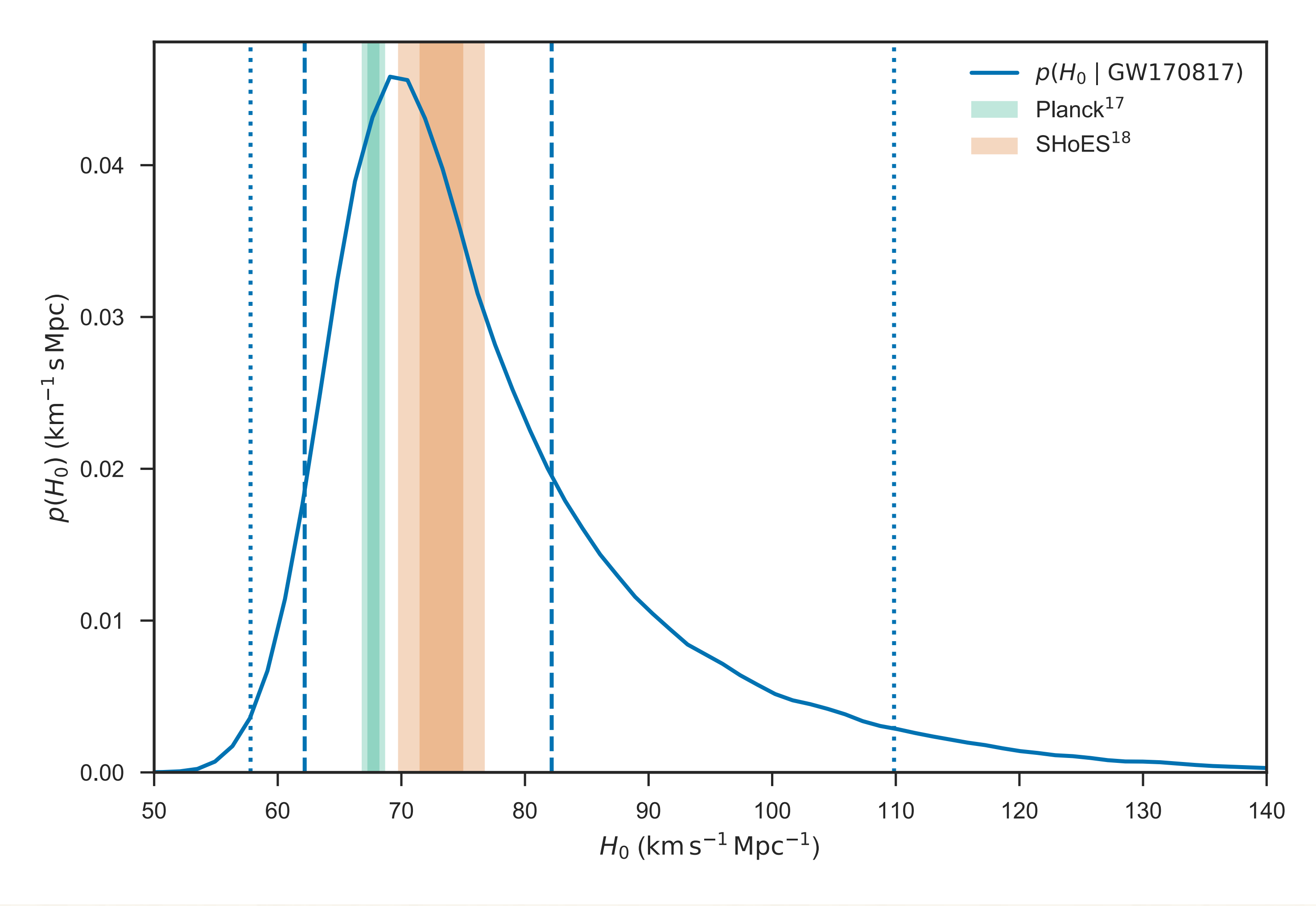
The Hubble Constant after GW170817

Jonathan Gair, Albert Einstein Institute Potsdam

From Einstein and Eddington to LIGO: 100 years of gravitational light deflection, Principe, May 28th 2019



Talk outline

- Eddington and Cosmology
- Eddington and Gravitational Waves
- GW170817
- Gravitational wave sources as cosmological probes
  - GW170817: first gravitational wave constraint on H_0;
  - statistical H_0 measurements with ground-based detectors;
  - prospects for improved cosmological measurements using future observations;
  - sources of systematics in GW constraints on cosmology.

Cosmological models

- Standard cosmological model starts with homogeneous and isotropic line element

$$\mathrm{d}s^2 = c^2\mathrm{d}\tau^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\Sigma^2, \qquad d\Sigma^2 = \frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

- and stress-energy tensor of perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$$

- Einstein's equations then yield the (Friedmann) equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi}{3}\rho$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \Lambda = -\frac{8\pi}{3}p$$

- The expansion rate $H = \dot{a}/a$ is called *the Hubble parameter* and its value today, *the Hubble constant*, is denoted H_0.

Einstein Static Universe

- Setting $p = 0$ (dust), $k = 1$ (closed Universe) and the conditions

$$\Lambda = \frac{1}{a^2} \qquad\qquad \rho = \frac{\Lambda}{4\pi}$$

- gives $\ddot{a} = \dot{a} = 0$ and therefore represents a static Universe with size (and mass) determined by the Cosmological constant $a = 1/\sqrt{\Lambda}$.
- This is the Einstein static Universe. Einstein favoured this model as it gives a Universe that is eternal and finite.
- Hubble's observation of the recession velocity of spiral nebulae in 1929 cast doubt on the static Universe model.

Eddington and Cosmology

- In the 1930, Eddington published a paper "*On the instability of Einstein's spherical world*".

On the Instability of Einstein's Spherical World.
By A. S. Eddington, F.R.S.

1. Working in conjunction with Mr. G. C. McVittie, I began some months ago to examine whether Einstein's spherical universe is stable. Before our investigation was complete we learnt of a paper by Abbé G. Lemaître * which gives a remarkably complete solution of the various questions connected with the Einstein and de Sitter cosmogonies. Although not expressly stated, it is at once apparent from his formulæ that the Einstein world is unstable—an important fact which, I think, has not hitherto been appreciated in cosmogonical discussions. Astronomers are deeply interested in these recondite problems owing to their connection with the behaviour of spiral nebulæ; and I desire to review the situation from an astronomical standpoint, although my original hope of contributing some definitely new result has been forestalled by Lemaître's brilliant solution.

5. *Instability of Einstein's Universe.*—Setting $p = 0$ in (4) we have

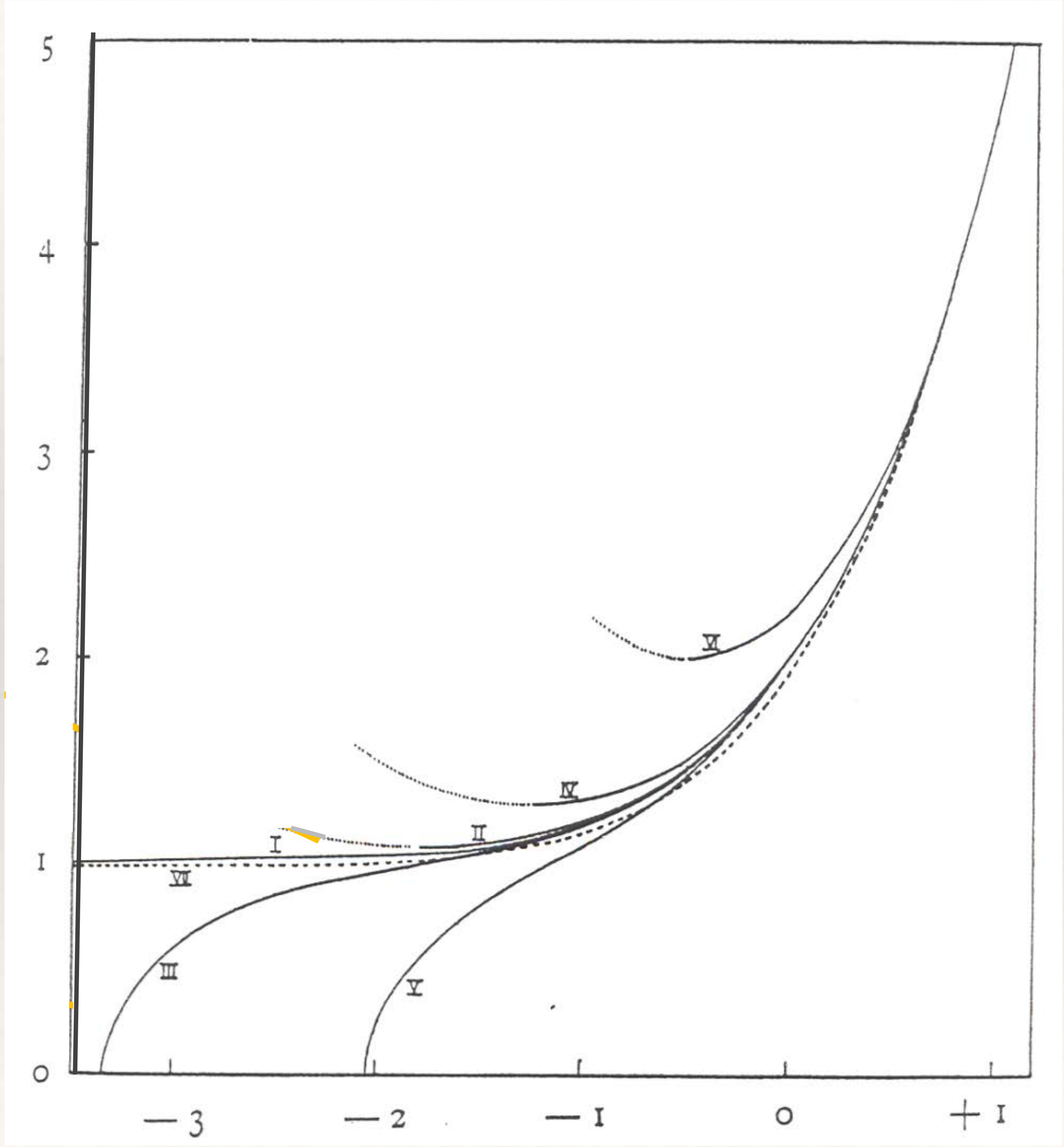
$$3\frac{d^2a}{dt^2} = a(\lambda - 4\pi\rho).$$

For equilibrium (Einstein's solution) we must accordingly have $\rho = \lambda/4\pi$. If now there is a slight disturbance so that $\rho < \lambda/4\pi$, d^2a/dt^2 is positive and the universe accordingly expands. The expansion will decrease the density; the deficit thus becomes worse, and d^2a/dt^2 increases. Similarly if there is a slight excess of mass a contraction occurs which continually increases. Evidently Einstein's world is unstable.

- In fact, Lemaitre (Eddington's student) had analysed cosmological solutions in 1927 implying similar conclusions, but Eddington had not read the paper at the time!

Eddington and Cosmology

- Eddington did not like a solution that starts at $a=0$, as *"The difficulty of applying this case is that it seems to require a sudden and peculiar beginning of things"*.
- Given Hubble's observations Eddington favoured a solution that was the static Universe asymptotically in the past, but now expanding.
- This model (the Eddington-Lemaitre model) was still considered viable in the 1990s, but it is difficult to reconcile with the CMB and high redshift objects (e.g., galaxy GN-z11 at z = 11.09).



Eddington and Cosmology

- While Eddington's favoured model is now ruled out, his 1930 paper made a number of important contributions
  - it popularised the notion that the Universe was expanding, and Eddington subsequently became a major advocate of this idea;
  - it introduced an analogy for the expanding Universe: *"It is as though they were embedded in the surface of a rubber balloon which is being steadily inflated."*
  - it stressed the naturalness of the existence of a cosmological constant *"on philosophical grounds"*;
  - it mentioned the (un-)importance of peculiar velocities: *"If the expansion during past history has been considerable we may expect the spiral nebulae to be nearly "at rest" so that the regular scattering apart will not be unduly masked by individual motions"*.
  - it contained a derivation of the gravitational redshift formula

$$\frac{a_0}{a} = 1 + z$$

Eddington and Gravitational Waves

- In 1922 Eddington wrote a paper entitled *"The Propagation of Gravitational Waves"*.
- The phrase *"the only speed of propagation relevant to them is the speed of thought"* is often mis-quoted as evidence Eddington did not believe in the physicality of GWs.

The Propagation of Gravitational Waves.

By A. S. Eddington, F.R.S.

(Received October 11, 1922.)

1. The problem of the propagation of disturbances of the gravitational field was investigated by Einstein in 1916, and again in 1918.* It has usually been inferred from his discussion that a change in the distribution of matter produces gravitational effects which are propagated with the speed of light; but I think that Einstein really left the question of the speed of propagation rather indefinite. His analysis shows how the co-ordinates must be chosen if it is desired to represent the gravitational potentials as propagated with the speed of light; but there is nothing to indicate that the speed of light appears in the problem, except as the result of this arbitrary choice. So far as I know, the propagation of the absolute physical condition—the altered curvature of space-time—has not hitherto been discussed.

Eddington and Gravitational Waves

- In fact, Eddington showed that transverse-transverse waves propagated at the speed-of-light in any coordinate system, while longitudinal-transverse and longitudinal-longitudinal waves traveled at arbitrary velocity - *"the speed of thought"*.

$$h_{22} + h_{33} = 0.$$

$$(1 - V^2)(h_{22}, h_{33}, h_{23}) = 0.$$

$$h_{24} = Vh_{12}\,;\ h_{34} = Vh_{13}.$$

$$h_{44} - 2Vh_{14} + V^2 h_{11} = 0.$$

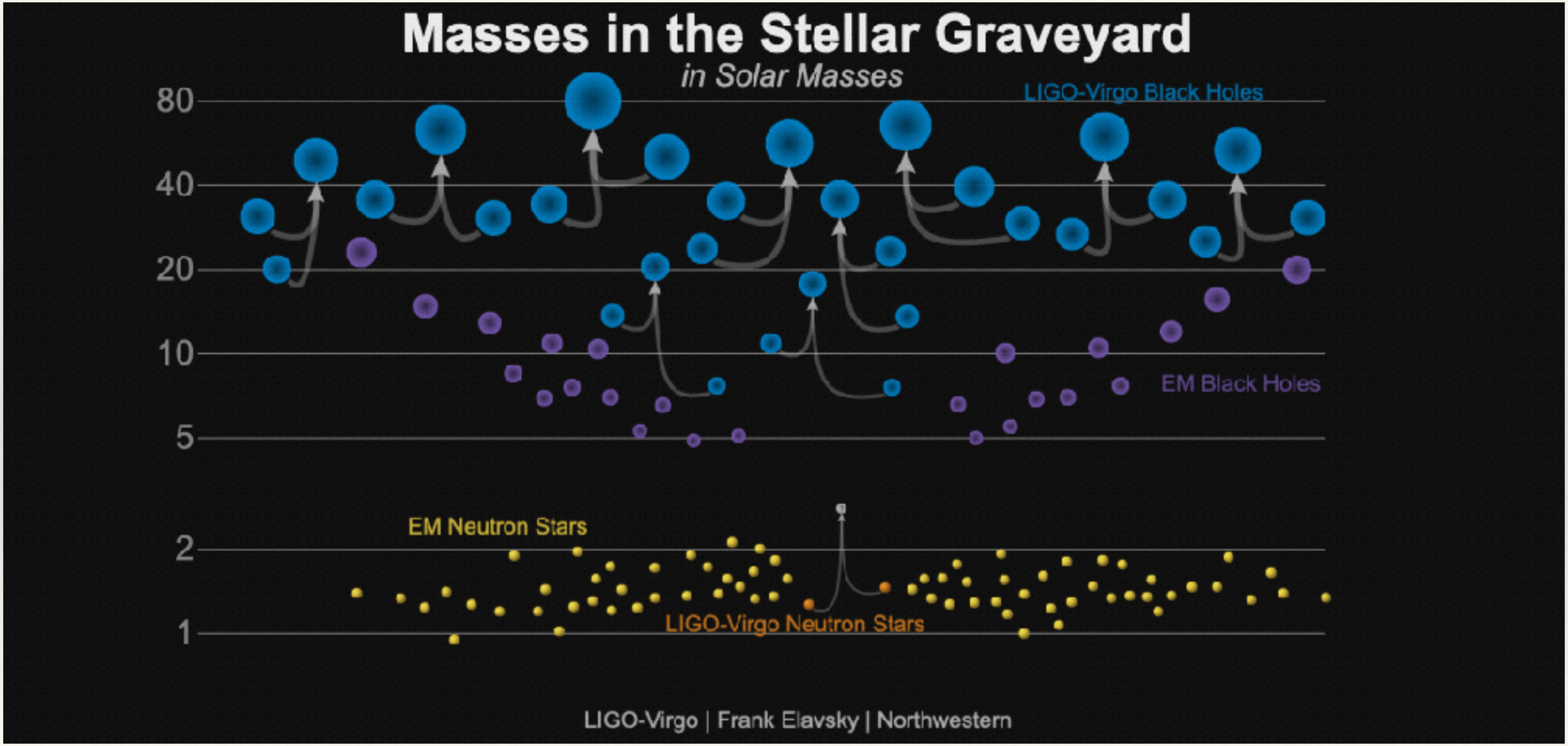
- He argued that the LT and LL modes were therefore not physical - these are gauge artefacts.
- Eddington showed that there are just two physical wave degrees of freedom, that these propagate at the speed of light and that they carry energy.

Gravitational Wave Detectors

Gravitational wave detectors

- A network of ground-based detectors is currently operating
  - LIGO: two 4km interferometers in Hanford, WA and Livingston, LA. Advanced LIGO began taking data in September 2015. O3 observing run currently underway.
  - Virgo: 3km interferometer at Cascina, near Pisa, Italy. Advanced Virgo began to collect data in late July 2017.
  - Japanese detector, KAGRA, (2019) and third LIGO detector in India (2024), to come online soon.

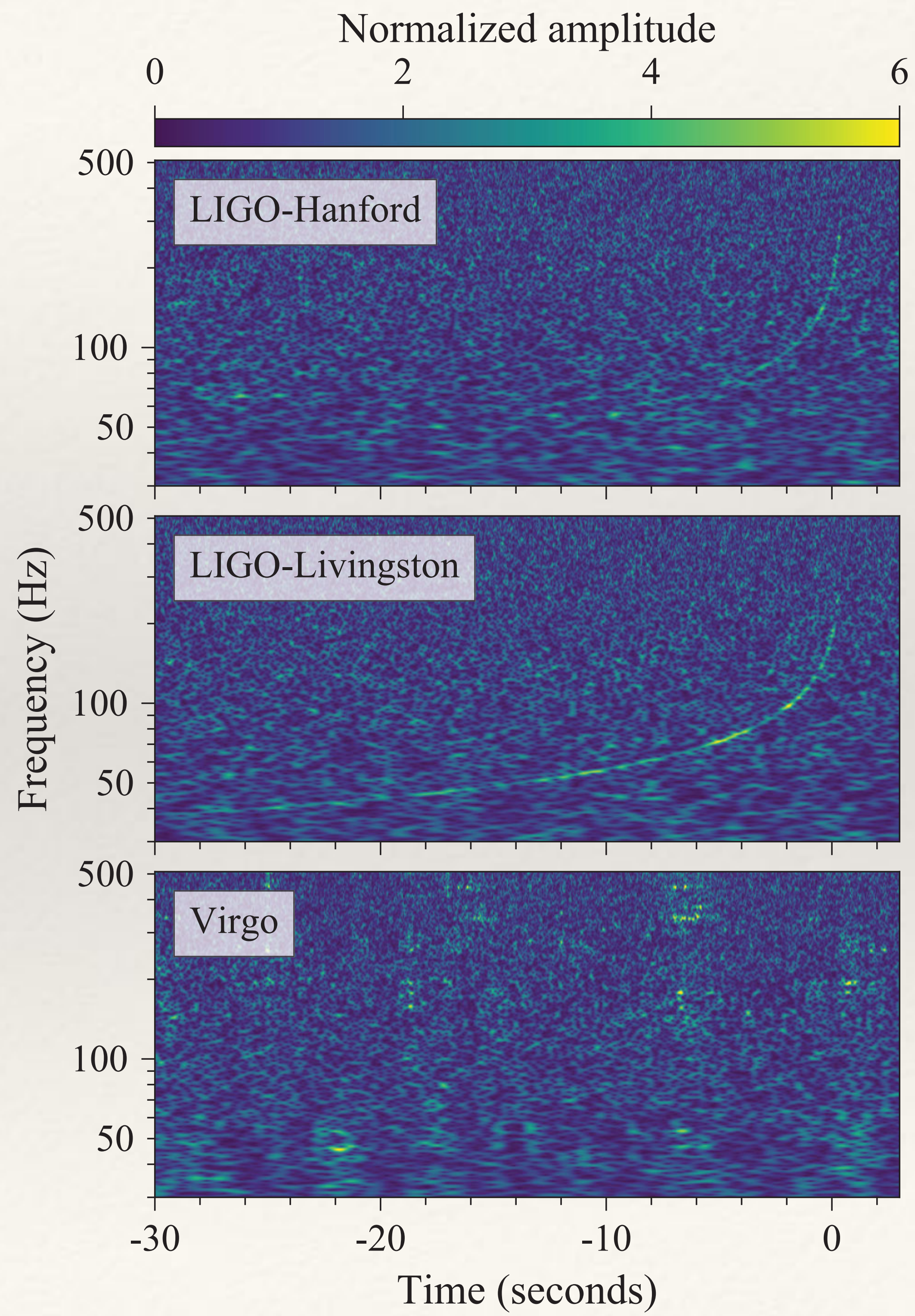
LVC Observations



GW170817

GW170817

- At 12:41:04 UTC on August 17th 2017, Advanced LIGO and Advanced Virgo made their first observation of a binary neutron star event.
- It was loud - network signal-to-noise ratio of 32.4 (loudest GW event to date).
- Estimated false alarm rate < 1 in 80,000 years.
- A coincident GRB was observed by Fermi, 1.7s after the merger time inferred from the GWs.

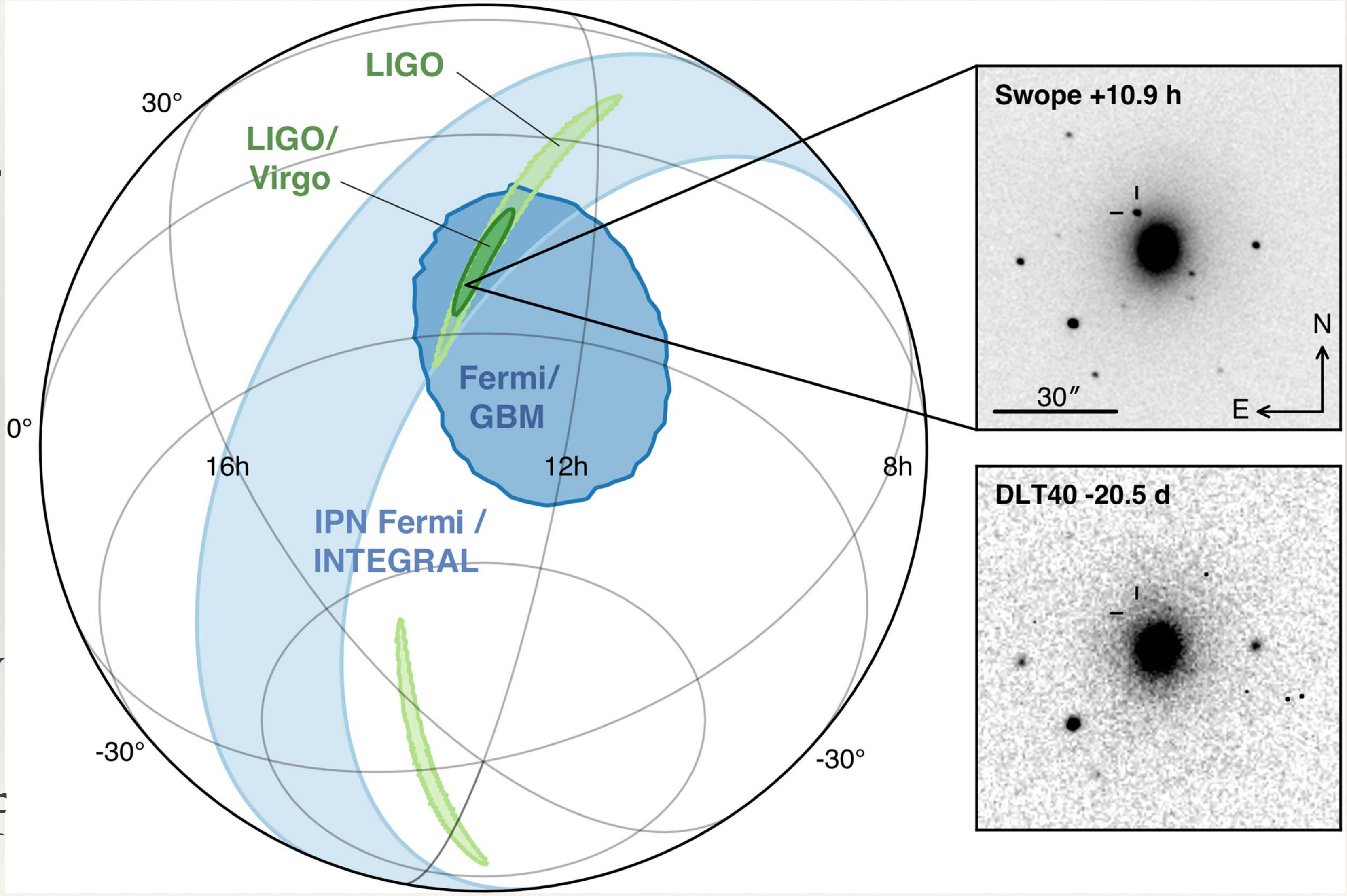


EM Follow-up

- LIGO issued an alert (GCN) to EM partners. It was followed-up by multiple groups that night and an optical counterpart found.

- The optical counterpart identified the host galaxy as NGC 4993, a galaxy in the constellation of Hydra at sky location ra=13h09m48s, dec = -23°22′53″.

- Multi-messenger observations facilitate many different science investigations, e.g., speed of gravitational waves.



LVC+, *Astrophys. J. Lett.* **848** L12 (2017)

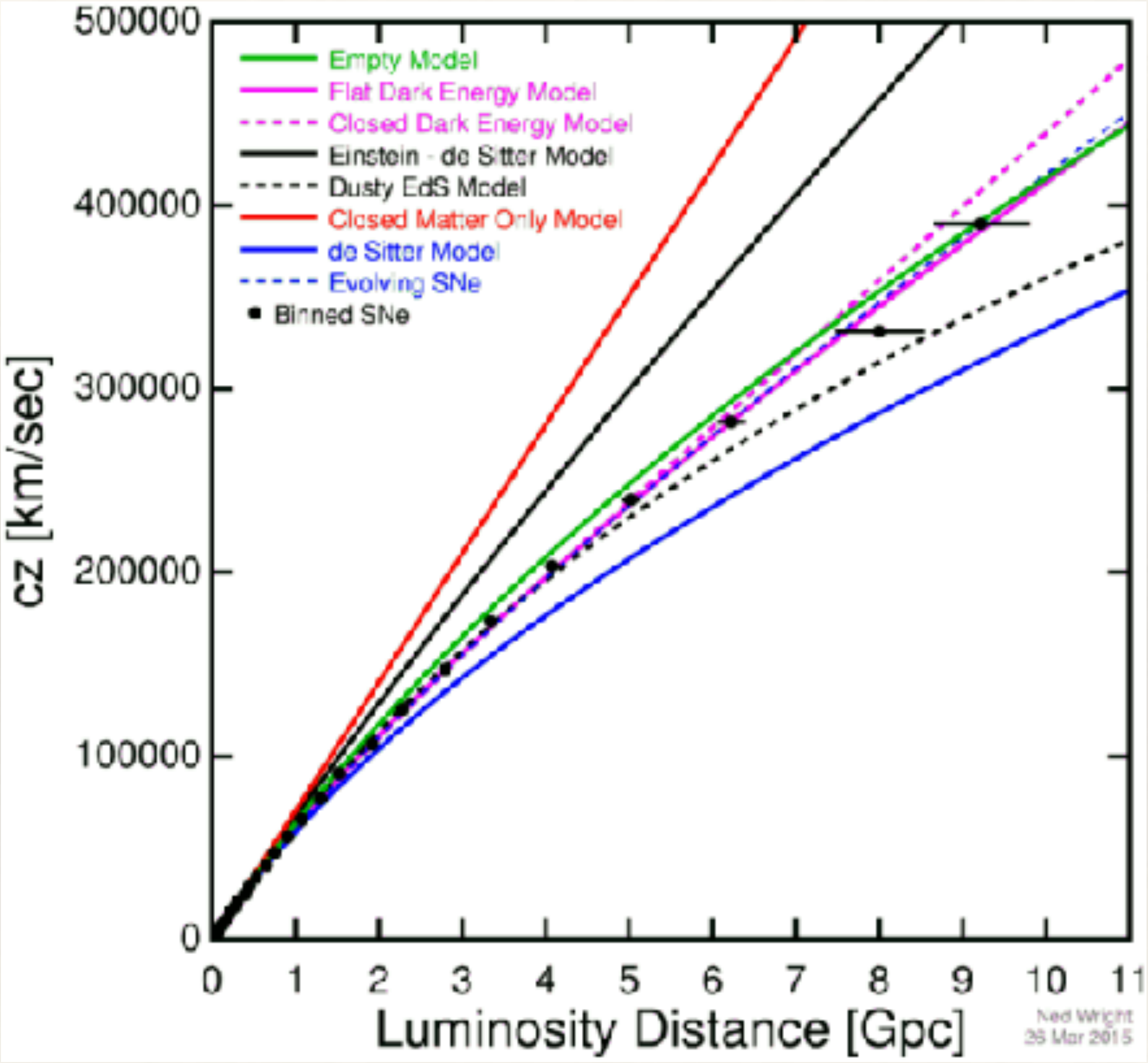
Measurements of H_0 using counterparts

Standard Sirens

- Basic idea: gravitational wave strain scales as

$$h_{+,\times} \sim f_{+,\times}(\cos\iota)\frac{\mathcal{M}}{\mathcal{D}} \sim f_{+,\times}(\cos\iota)\frac{(1+z)M}{D_L(z)}$$

- Phase evolution determines intrinsic parameters (e.g., mass) to high accuracy. Amplitude then determines distance (Schutz 1986).
- If redshift can be obtained, we get a point on the D_L-z relationship
- Problem is to obtain redshift. Three methods: counterpart, statistical or by assuming a mass function.



Plot from Betoule et al. (2004)

Hubble constant measurement

- Redshift of NGC 4993, $v_{\rm rec} = 3327 \pm 72\,{\rm km\,s^{-1}}$, and GW distance of GW170817, $d = 43.8^{+2.9}_{-6.9}$ Mpc, gives $H_0 = v_{rec}/d = 76.0$ km s^{-1} Mpc^{-1}.
- There are two potential complications:
    - 1) NGC 4993 is sufficiently close that it has a significant peculiar velocity;
    - 2) selection effects in gravitational wave and electromagnetic measurements.

Peculiar velocity correction

- Can map peculiar velocity field using Fundamental Plane relation for galaxy properties. In this way Springlob et al. find $v_p = 310 \pm 69 \mathrm{km\,s}^{-1}$ for NGC 4993 using 6df redshift catalogue.

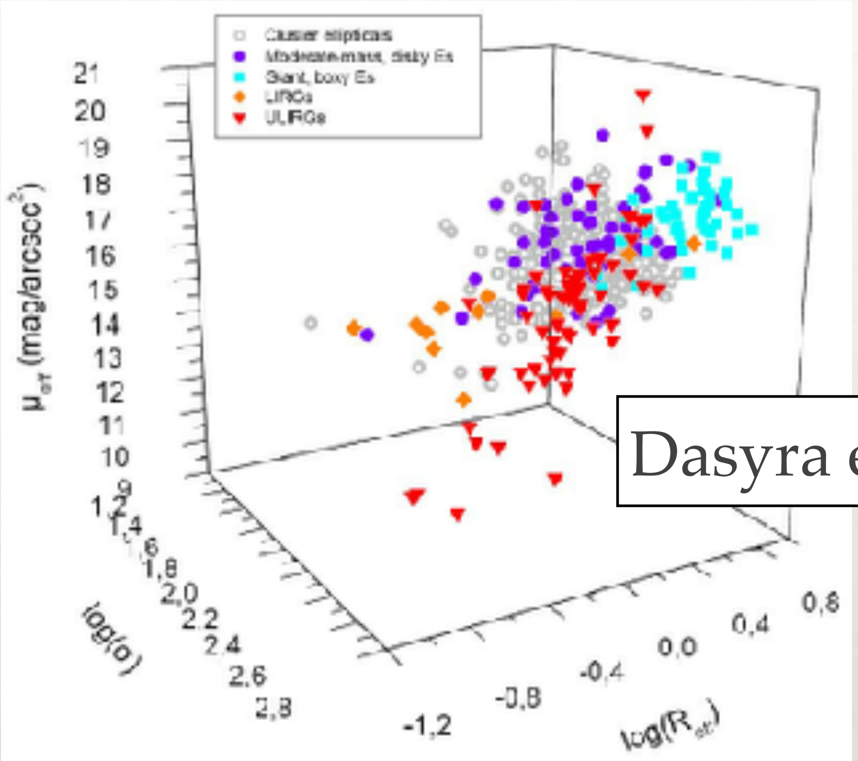
- Can also use Tully-Fisher relation. Using 2MASS redshift survey, Carrick et al. (2015) find $v_p = 280 \pm 150 \mathrm{km\,s}^{-1}$

- We adopted the value

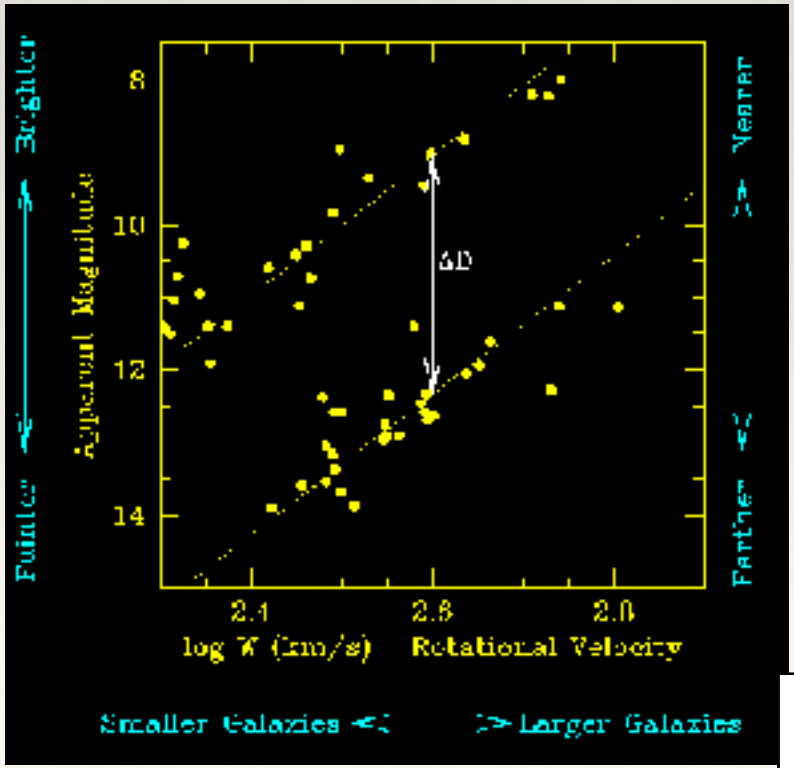
$$v_p = 310 \pm 150 \mathrm{km\,s}^{-1}$$

- so we effectively used

$$cz_H = v_H = 3017 \pm 166 \mathrm{km\,s}^{-1}$$



Dasyra et al. (2018)



Credit: NOAO

Hubble constant measurement

- The likelihood for the observed data is

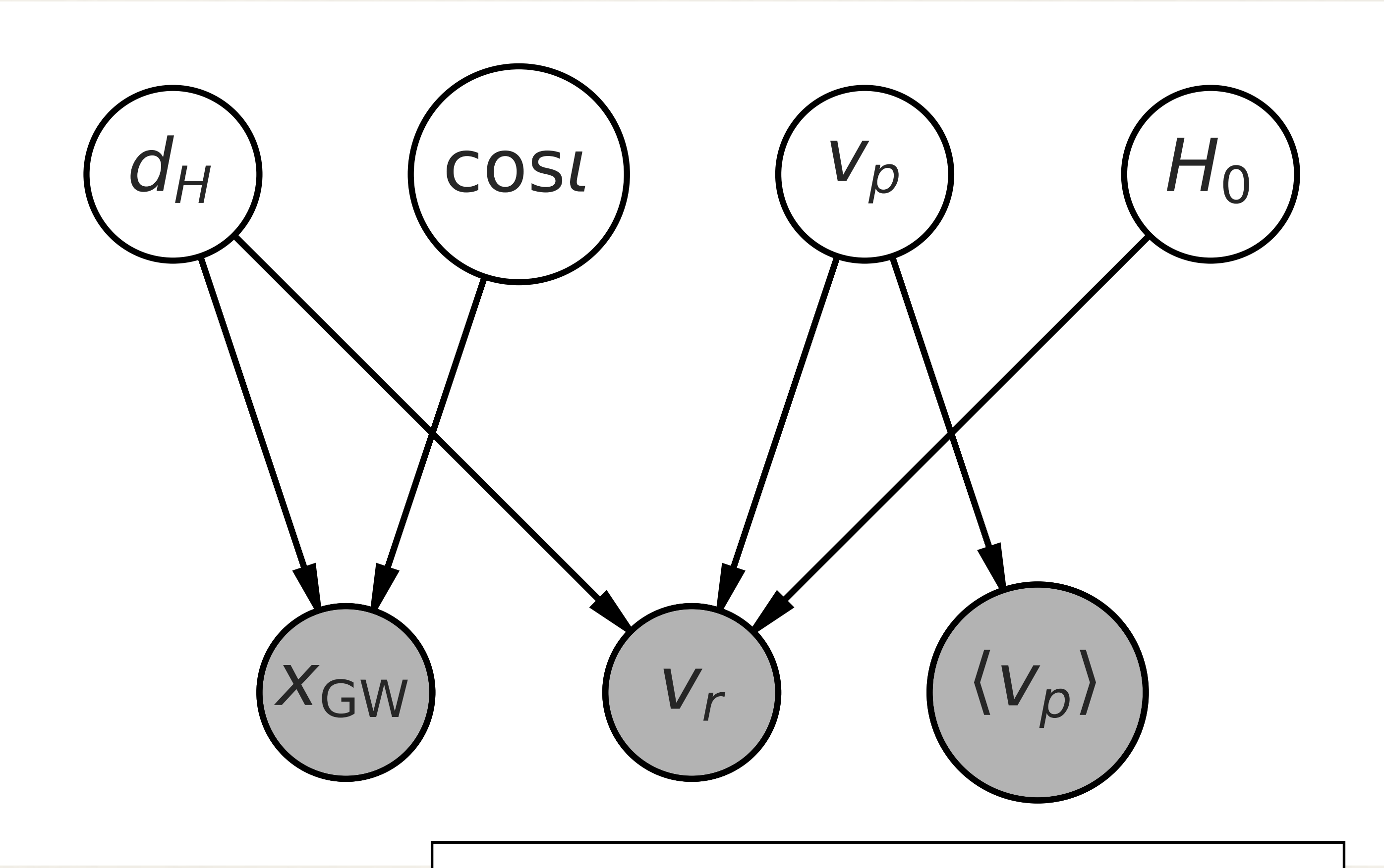
$$p(x_{\rm GW}, v_r, \langle v_p\rangle \mid d, \cos\iota, v_p, H_0) =$$
$$p(x_{\rm GW} \mid d, \cos\iota)\, p(v_r \mid d, v_p, H_0)\, p(\langle v_p\rangle \mid v_p)$$

- with

$$p\left(v_r \mid d, v_p, H_0\right) = N\left[v_p + H_0 d, \sigma^2_{v_r}\right](v_r)$$
$$p\left(\langle v_p\rangle \mid v_p\right) = N\left[v_p, \sigma^2_{v_p}\right](\langle v_p\rangle)$$

- Giving the posterior on H_0

$$p(H_0, d, \cos\iota, v_p \mid x_{\rm GW}, v_r, \langle v_p\rangle)$$
$$\propto \frac{p(H_0)}{\mathcal{N}_{\rm s}(H_0)}\, p(x_{\rm GW} \mid d, \cos\iota)\, p(v_r \mid d, v_p, H_0)$$
$$\times\ p(\langle v_p\rangle \mid v_p)\, p(d)\, p(v_p)\, p(\cos\iota),$$



LVC+, *Nature Lett.* **551** 85 (2017)

Hubble constant measurement

- Selection effects are encoded in

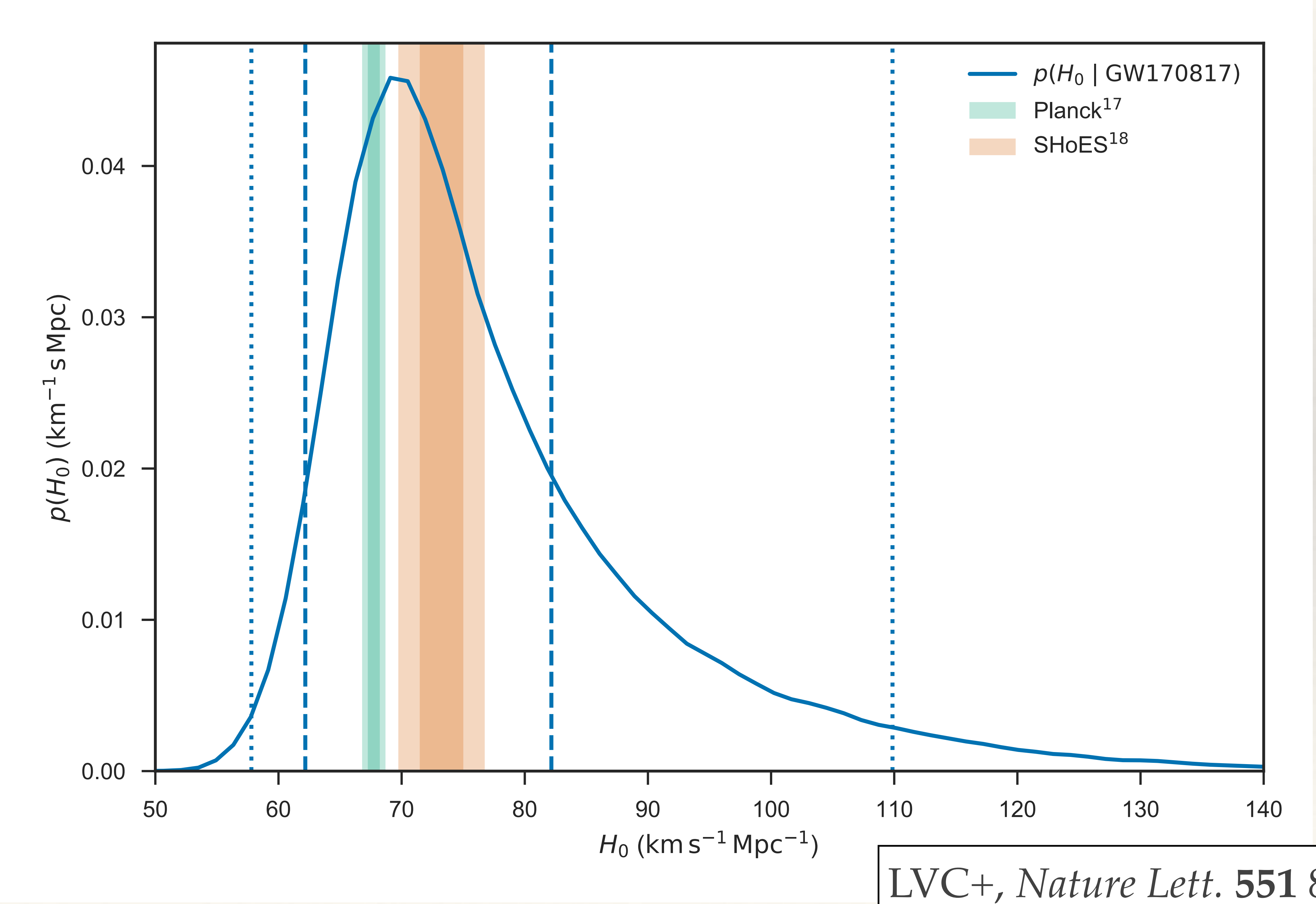
$$\mathcal{N}_\mathrm{s}(H_0) = \int_{\text{detectable}} \mathrm{d}\vec{\lambda}\,\mathrm{d}d\,\mathrm{d}v_p\,\mathrm{dcos}\,\iota\,\mathrm{d}x_\mathrm{GW}\,\mathrm{d}v_r\,\mathrm{d}\langle v_p\rangle$$

$$\times\left[p(x_\mathrm{GW}\mid d,\cos\iota,\vec{\lambda})\,p(v_r\mid d,v_p,H_0)\times p(\langle v_p\rangle\mid v_p)\,p(\vec{\lambda})\,p(d)\,p(v_p)\,p(\cos\iota)\right]$$

- Integral is over all data sets, $\{x_\mathrm{GW}, v_\mathrm{r}, <v_\mathrm{p}>\}$, that would be analysed.
- At time of GW170817, GW selection effects dominated
  - BNS horizon for LIGO-Virgo network: ~190Mpc;
  - EM counterpart ~17mag in I band. Still easily detectable at 400Mpc (~22 mag).
- GW selection is on signal to noise. This depends directly on distance (no H_0 dependence). Weak redshift dependence only through differential redshifted mass sensitivity.
- Selection effects will be important when LIGO horizon increases, for sources at cosmological distances and for statistical analyses using redshift catalogues.

Hubble constant measurement

- Final result is $H_0 = 70.0^{+12.0}_{-8.0}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$



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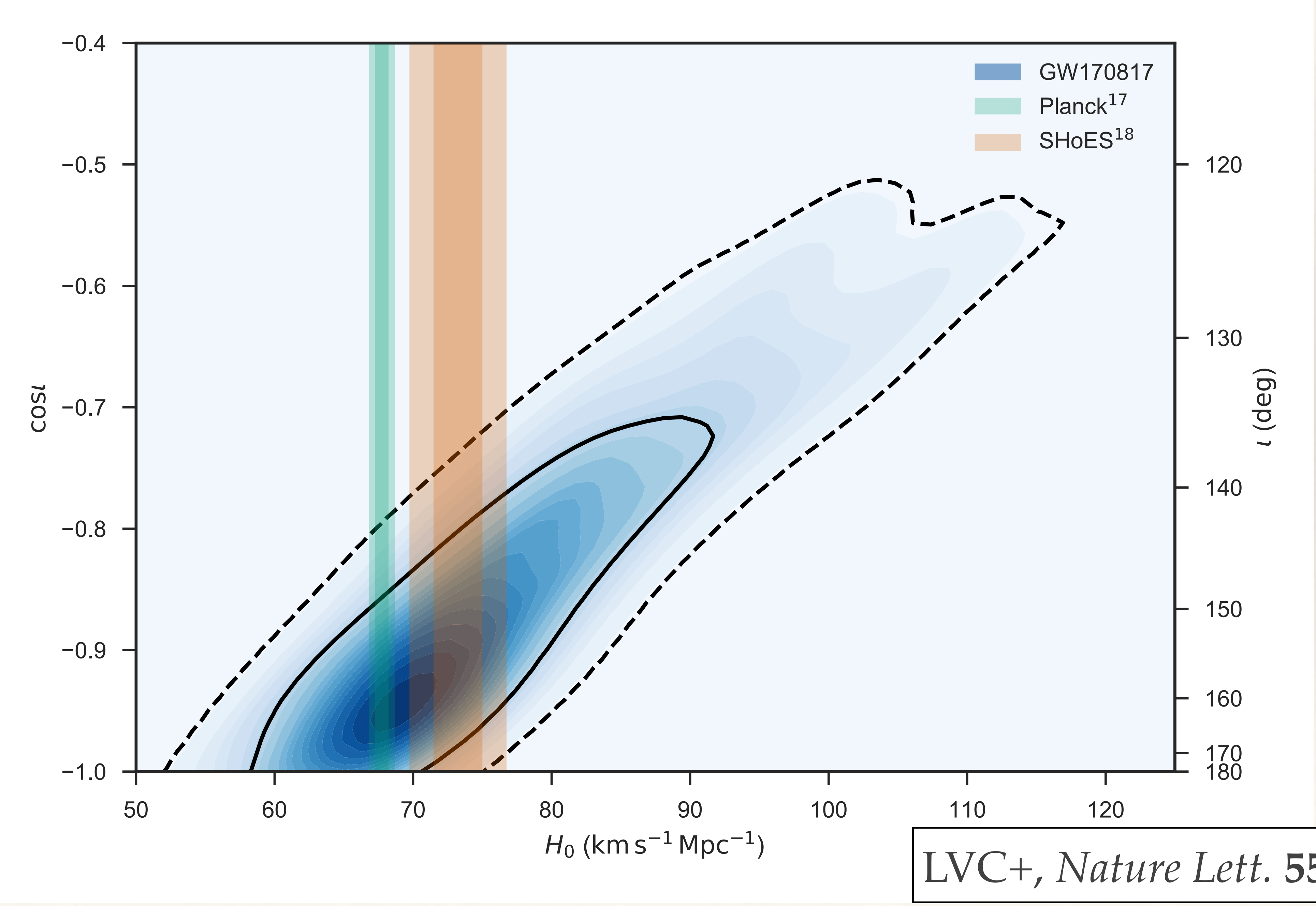
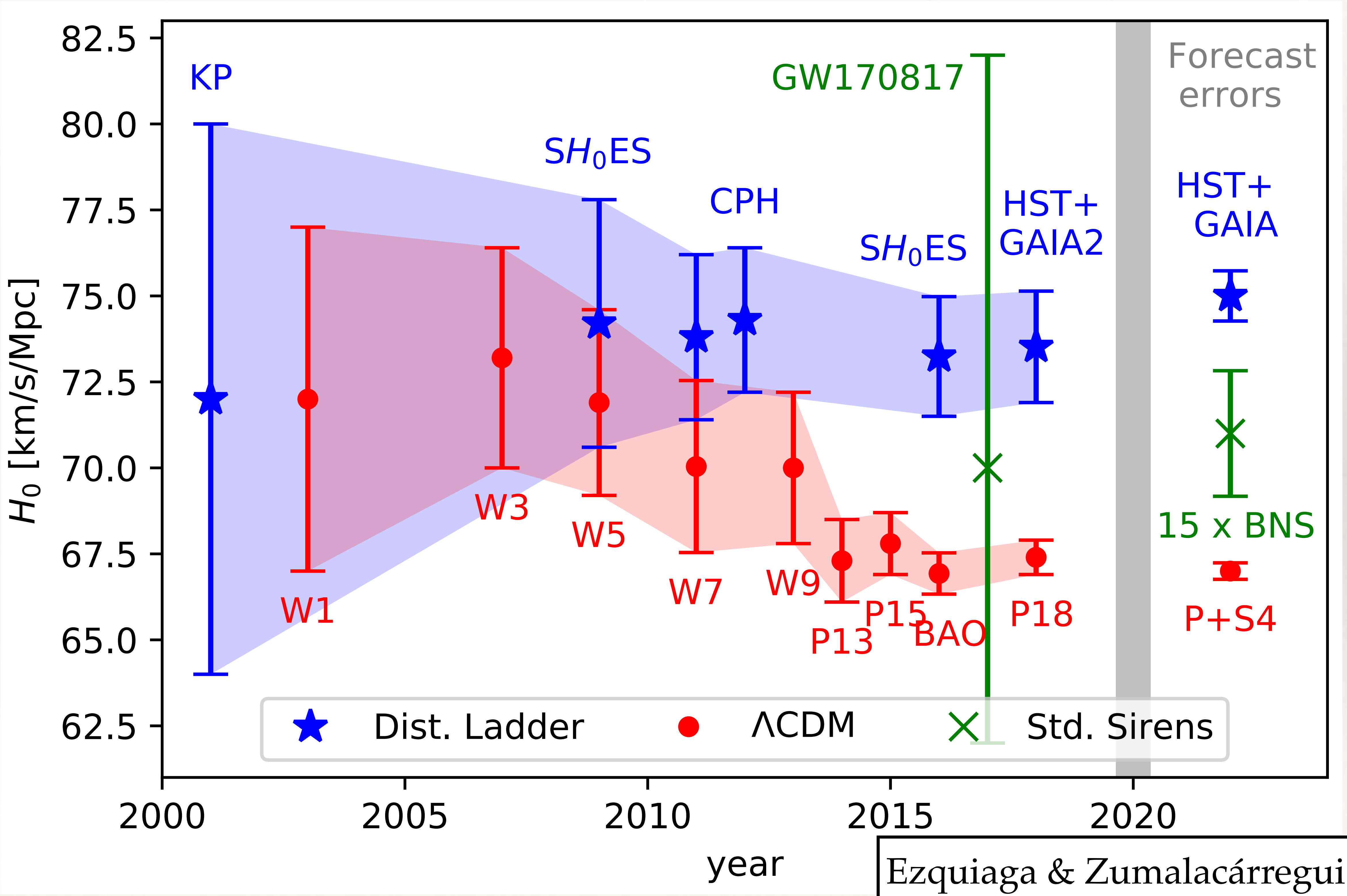
H_0-inclination degeneracy

- Uncertainty in measurement largely driven by degeneracy between distance and inclination of the source.



LVC+, *Nature Lett.* **551** 85 (2017)

Current H_0 Tension



Ezquiaga & Zumalacárregui 2018

Prospects for future measurements: counterpart case

Future H_0 measurements: counterparts

- If we optimally combine a set of observations of the form $N(\mu, \sigma_i^2)$ to estimate the common mean, then after n observations the uncertainty in the mean is $\Sigma/\sqrt{n}$ where
$$\frac{1}{\Sigma^2} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\sigma_i^2}$$
- Hence we can approximate Σ^2 by $1 \ / \ \mathbb{E}(1/\sigma_i^2)$
- Measurement precision scales with SNR as $\sigma_i \propto 1/\rho$ and SNR scales with distance as $\rho \propto 1/d$. Hence, for a uniform in comoving volume population we have an SNR distribution $p(d) \propto d^2 \Rightarrow p(\rho) = 3\rho_{\rm th}^3/\rho^4$
- For GW170817 the SNR was 32.4 and uncertainty ~7km s^{-1} Mpc^{-1}. Threshold SNR is approximately 12. Hence we can estimate
$$\Sigma \approx \frac{\sigma_{GW170817}\rho_{GW170817}}{\sqrt{3}\rho_{\rm th}} \approx 10.9$$

Future H_0 measurements: counterparts

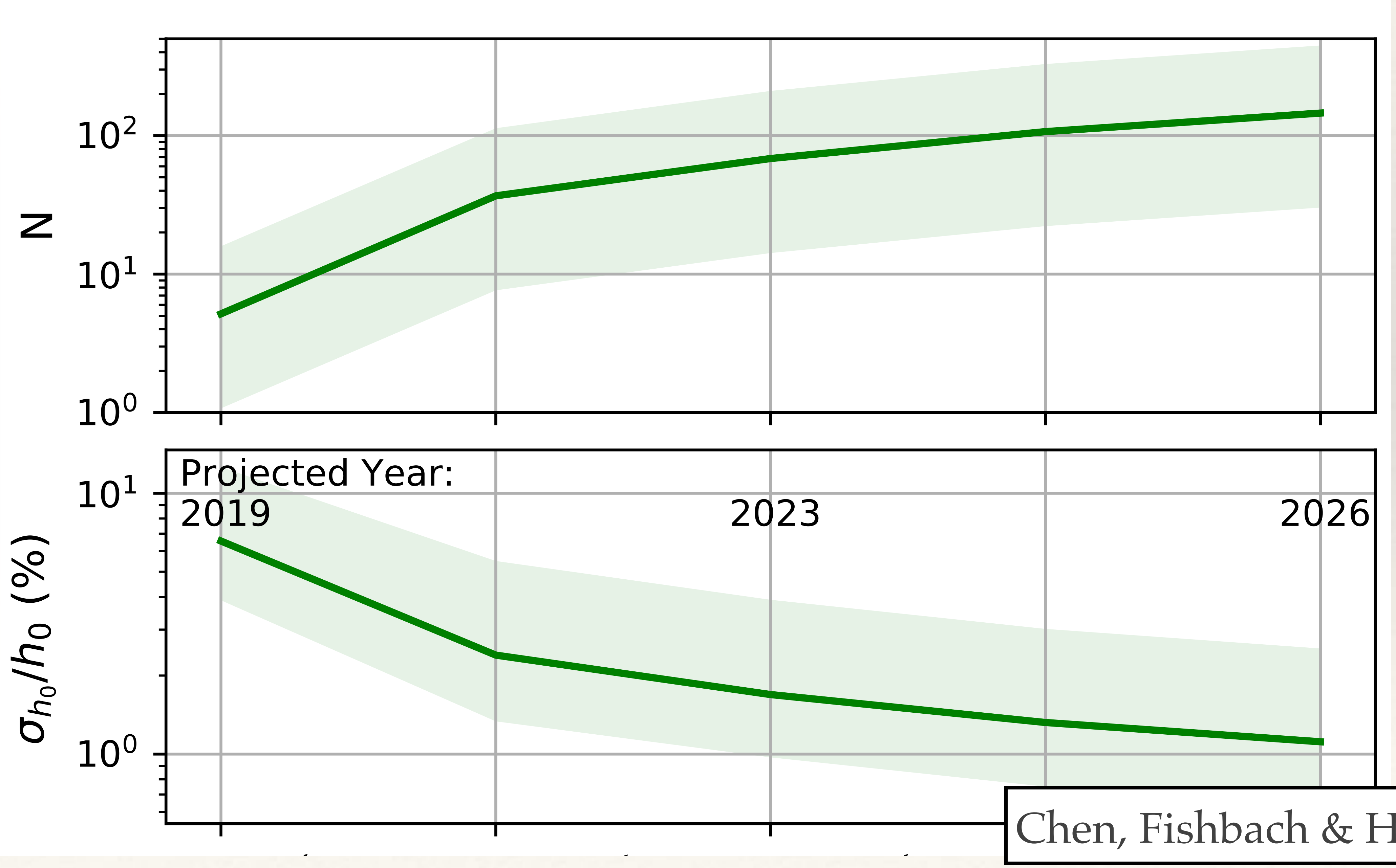
- To distinguish Planck and SHoES need precision better than (73.24 - 67.74)/4 -> 2% measurement of H_0. This require n > 60 events.
- For a 1% H_0 measurement, need n > 240 events.

Epoch		2015 – 2016	2016 – 2017	2018 – 2019	2020+	2024+
Planned run duration		4 months	9 months	12 months	(per year)	(per year)
Expected burst range/Mpc	LIGO	40 – 60	60 – 75	75 – 90	105	105
	Virgo	—	20 – 40	40 – 50	40 – 70	80
	KAGRA	—	—	—	—	100
Expected BNS range/Mpc	LIGO	40 – 80	80 – 120	120 – 170	190	190
	Virgo	—	20 – 65	65 – 85	65 – 115	125
	KAGRA	—	—	—	—	140
Achieved BNS range/Mpc	LIGO	60 – 80	60 – 100	—	—	—
	Virgo	—	25 – 30	—	—	—
	KAGRA	—	—	—	—	—
Estimated BNS detections		0.05 – 1	0.2 – 4.5	1 – 50	4 – 80	11 – 180
Actual BNS detections		0	1	—	—	—
90% CR	% within 5 deg^2	< 1	1 – 5	1 – 4	3 – 7	23 – 30
	% within 20 deg^2	< 1	7 – 14	12 – 21	14 – 22	65 – 73
	median/deg^2	460 – 530	230 – 320	120 – 180	110 – 180	9 – 12
Searched area	% within 5 deg^2	4 – 6	15 – 21	20 – 26	23 – 29	62 – 67
	% within 20 deg^2	14 – 17	33 – 41	42 – 50	44 – 52	87 – 90

LVC, Liv. Rev. Rel. **19** 1 (2016)

Future H_0 measurements: counterparts

- These scalings are completely consistent with more careful simulations for a three detector network. Addition of KAGRA and LIGO India improves precision per event by ~15%.



Chen, Fishbach & Holz (2018)

Statistical measurements of H_0

Future H_0 measurements: statistical

- General form of likelihood is

$$p(H_0|\mathbf{d}^{GW},\mathbf{d}^{EM},\det) \propto \frac{p_h(H_0)}{\mathcal{N}_s(H_0)}\int p(\mathbf{d}^{GW},\mathbf{d}^{EM}|z_t,H_0,\vec{\lambda})p_z(z_t)\mathrm{d}z_t\mathrm{d}\vec{\lambda}$$

- If do not have EM data, prior on $p_z(z_t)$ important. Inference requires combining multiple observations, using

$$p(H_0|\{\mathbf{d}^{GW},\det\}) \propto \frac{p_h(H_0)}{(\mathcal{N}_s(H_0))^{N_{\rm obs}}}\prod_{i=1}^{N_{\rm obs}}\left[\int p_z(z_t)p(\vec{\lambda})p_{GW}(\mathbf{d}_i^{GW}|cz_t/H_0,z_t,\vec{\lambda})\mathrm{d}\vec{\lambda}\mathrm{d}z_t\right]$$

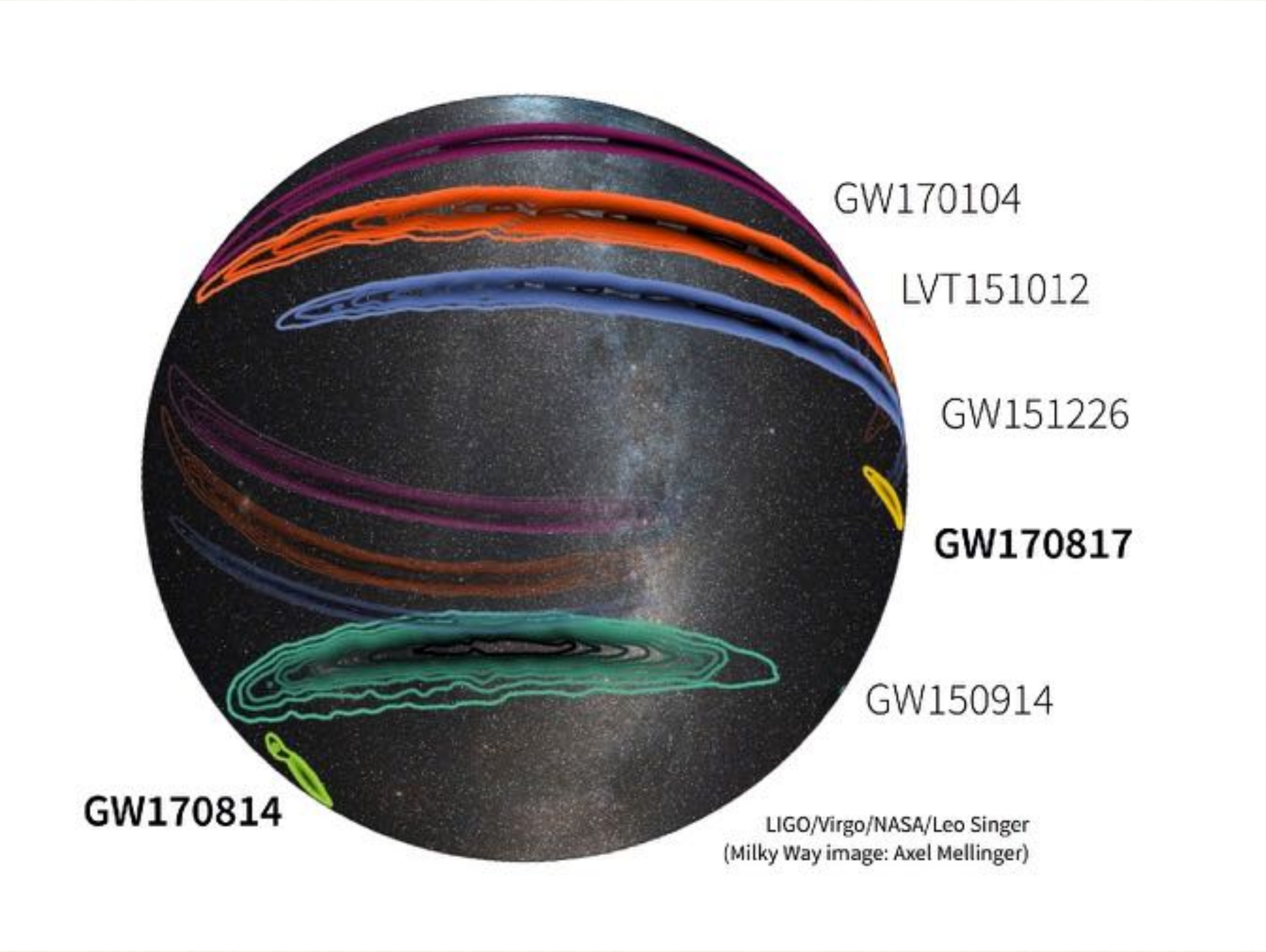
- Can construct $p_z(z_t)$ from a galaxy catalogue, but need to handle incompleteness of catalogue.

$$p_z(z) = p_{\rm cat}\sum_i \delta(z-z_i) + \frac{1}{V_c(z_{\rm max})}(1-f(z|H_0))\frac{\mathrm{d}V_c}{\mathrm{d}z}$$

$$f(z|H_0) = \int_{M_{\rm th}(z,H_0,m_{\rm th})}^{\infty} p(M|I)\mathrm{d}M$$

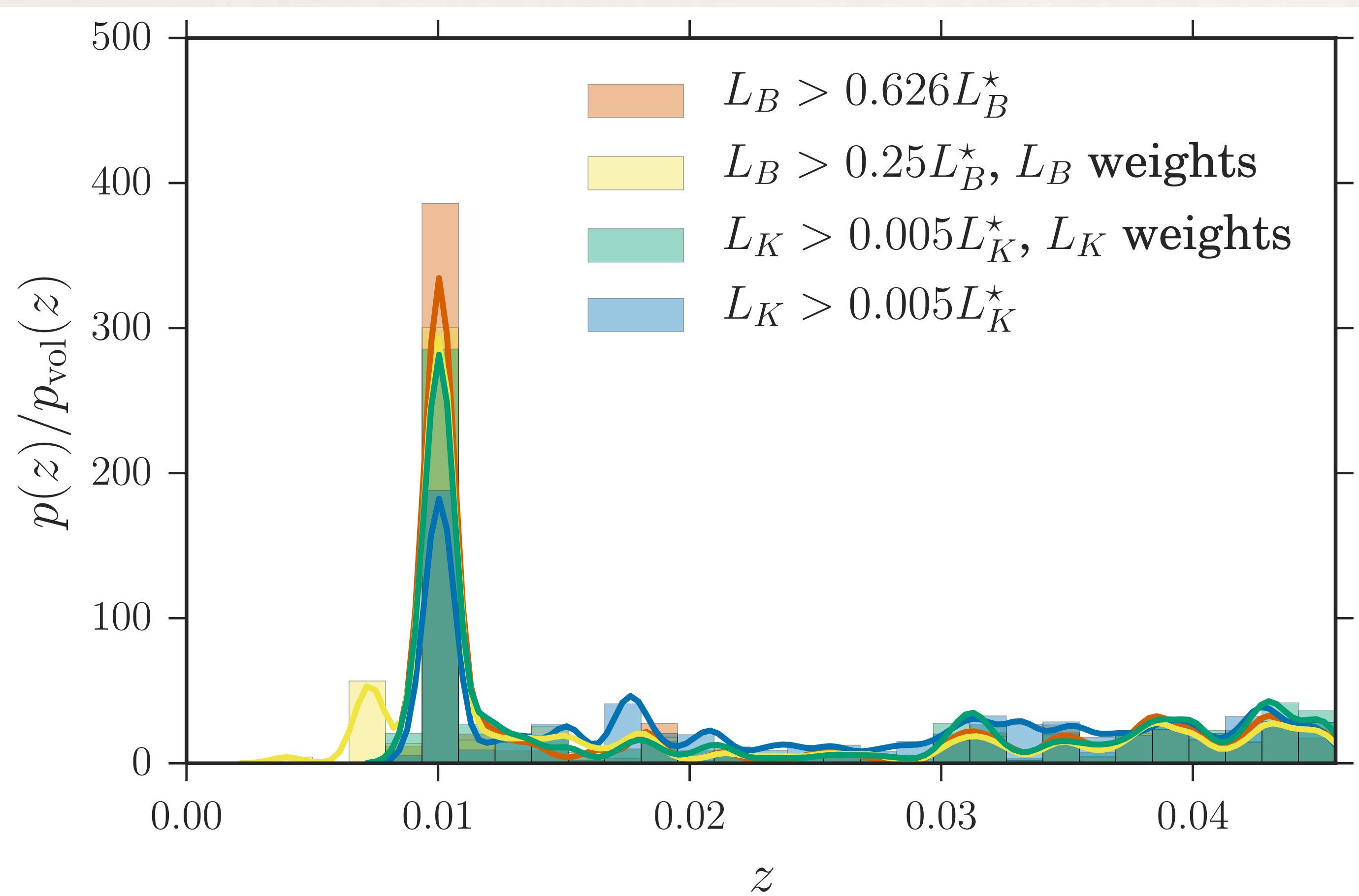
Future H_0 measurements: statistical

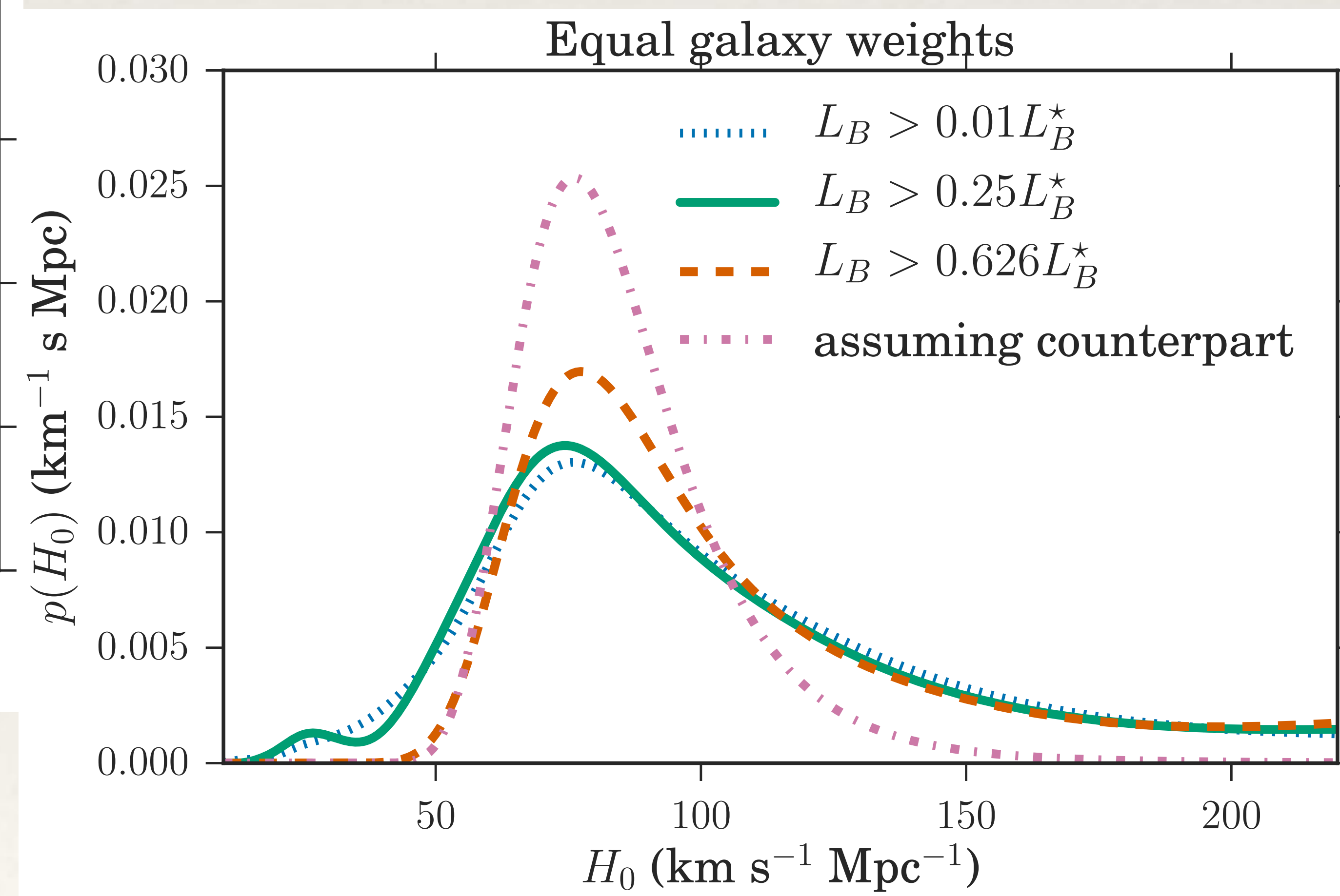
- Need small uncertainties for maximum power - post-Virgo events.



First statistical measurements: GW170817

- Carried out proof of principle of this measurement using GW170817. More informative than average since very close, but statistical measurement weaker than counterpart measurement.

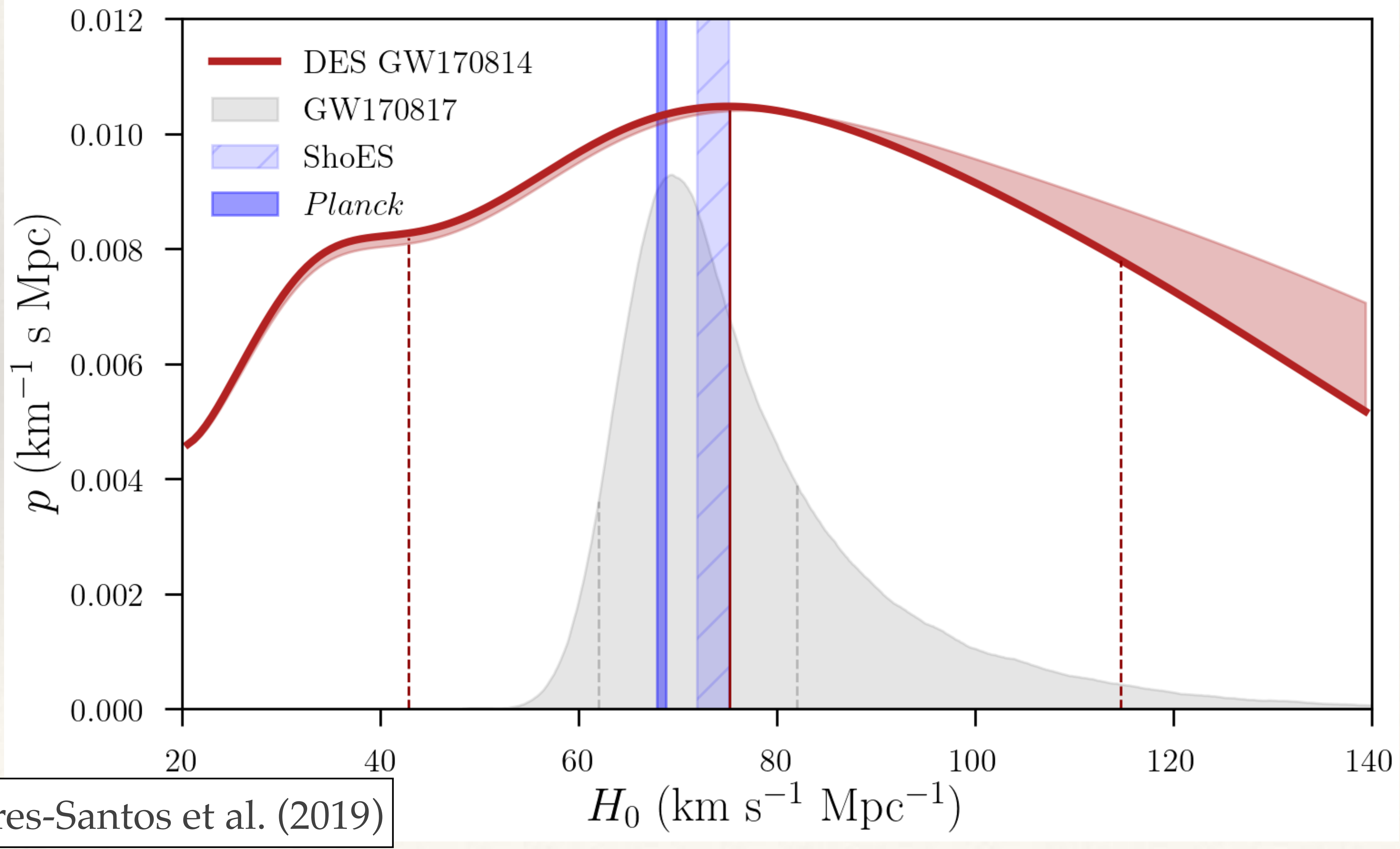




Fishbach, …, **JG**+ (2018)

First statistical measurements: GW170814

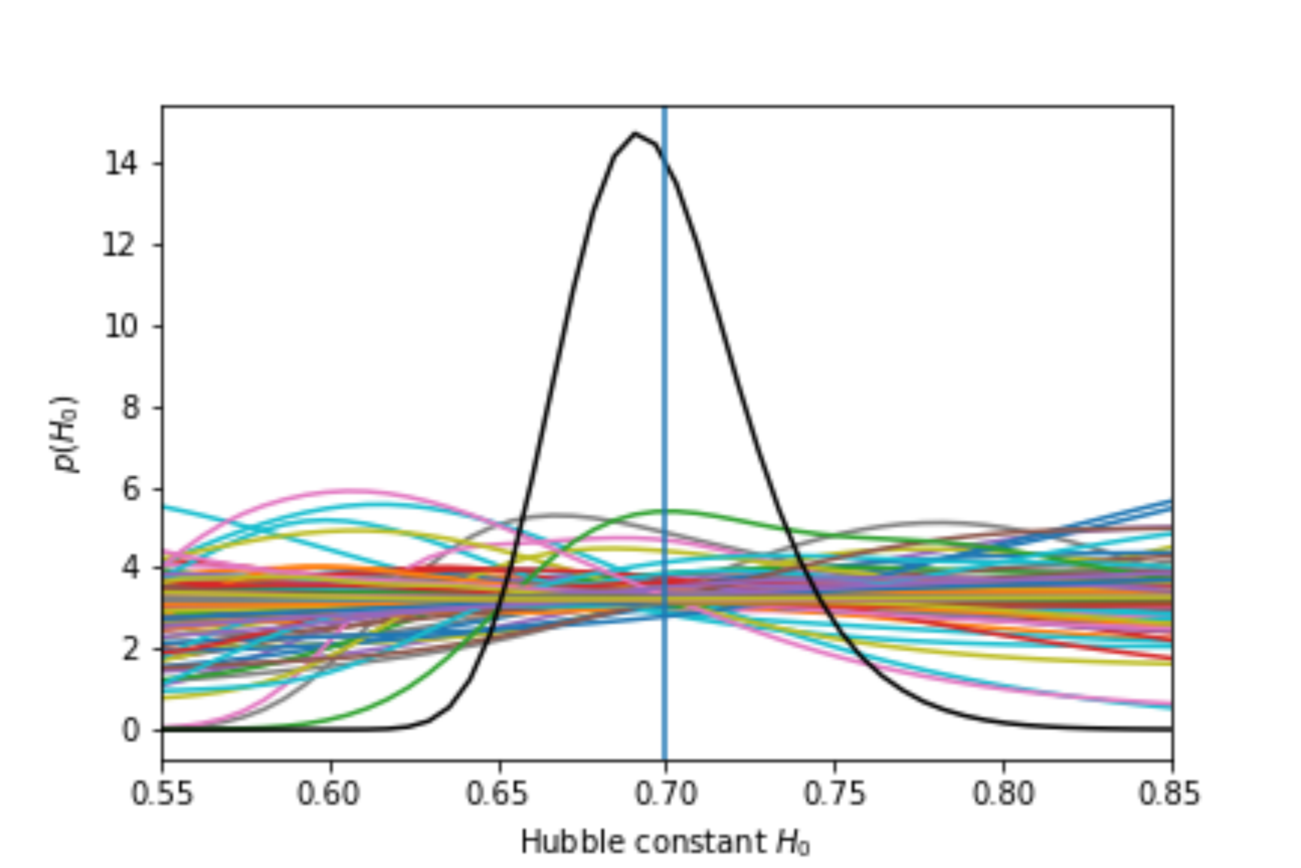
- Also applied to GW170814 by DES collaboration + LVC. First measurement using a dark siren.

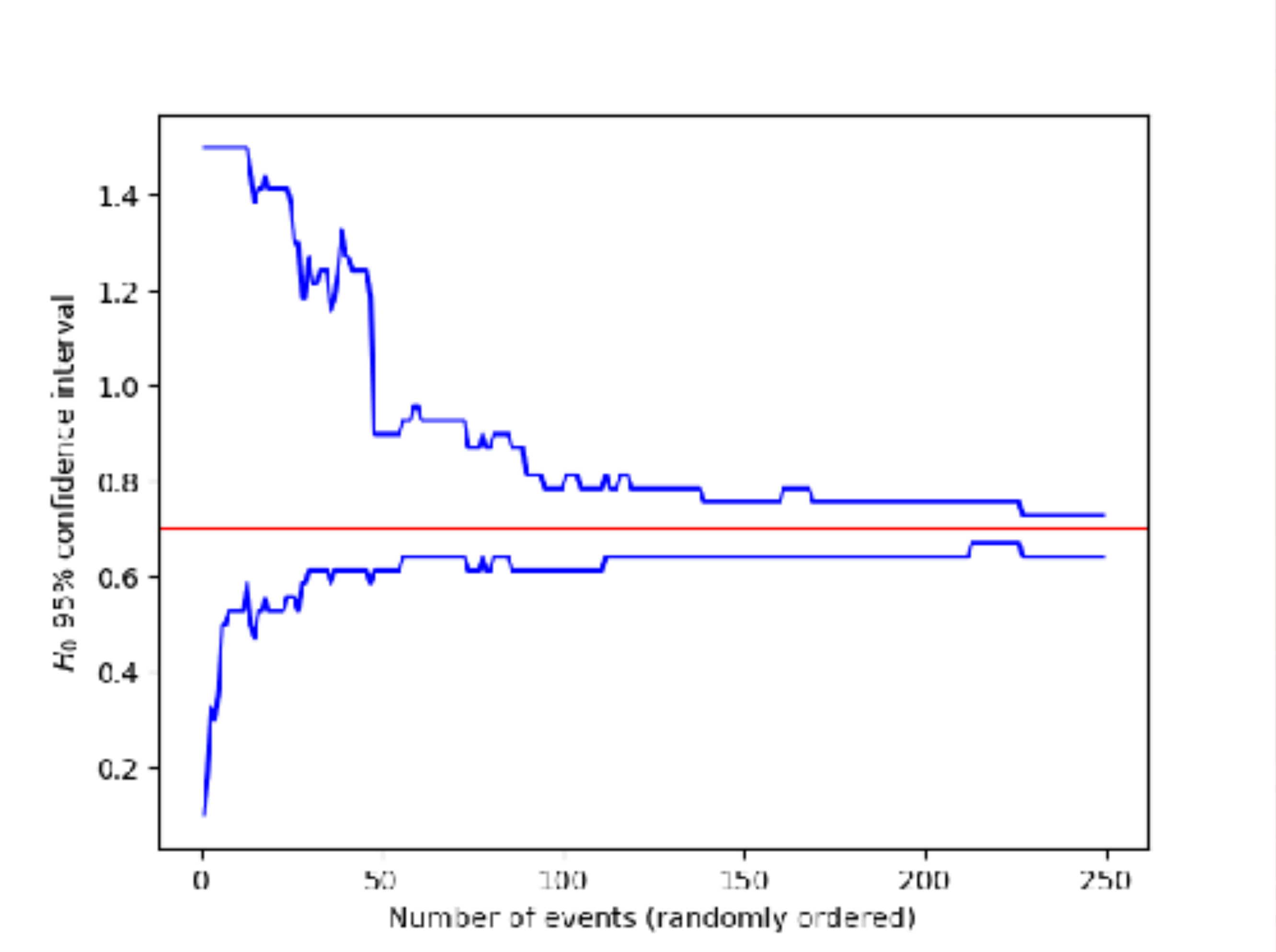


Soares-Santos et al. (2019)

Future H_0 measurements: statistical

- After combining 250 events, uncertainty is ~3km s^{-1} Mpc^{-1}. This would be achieved with ~15 counterpart events.





Plots produced by R. Gray

Systematic effects in Gravitational Wave Measurements of H_0

General statistical framework

- Considering all EM and GW observations together, general framework can be written

$$p(\{\mathbf{d}^{GW,\mathrm{det}}\}, \{\mathbf{d}^{EM,\mathrm{obs}}\}, \{\mathbf{d}^{EM,\mathrm{nobs}}\}, \{z\}^{\mathrm{nobs}}, \{z\}^{\mathrm{obs}}, \{\vec{\Omega}\}, N_{\mathrm{gal}}, \{\vec{\alpha}\}, \{\vec{\lambda}\}|\vec{\theta})$$

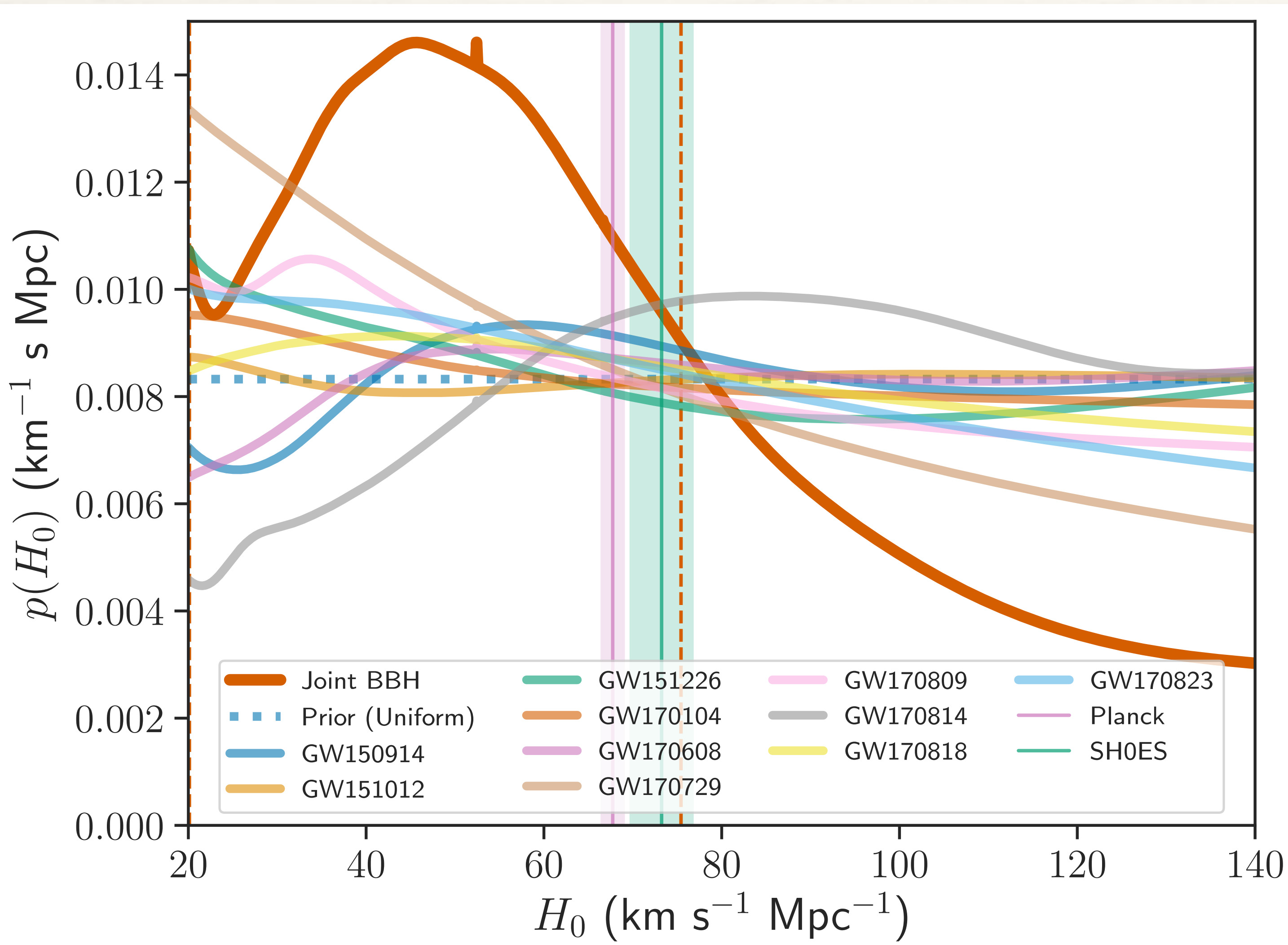
$$\propto \prod_{i=1}^{N_{GW}} \left[\frac{1}{p_{\mathrm{det}}^{GW}(\{z\}, \{\vec{\Omega}\}, \{\vec{\lambda}\}, \vec{\theta})} \frac{1}{\sum_{j=1}^{N_{\mathrm{gal}}} w(\vec{\lambda}_j, z_j, \vec{\Omega}_j)} \sum_{k=1}^{N_{\mathrm{gal}}} p(\mathbf{d}_i^{GW}, \mathbf{d}_i^{EM,\mathrm{new}}|\vec{\alpha}_i, z_k, \vec{\theta}) w(\vec{\lambda}_k, z_k, \vec{\Omega}_k) p(\vec{\alpha}_i|\vec{\lambda}_k, z_k, \vec{\Omega}_k) \right]$$

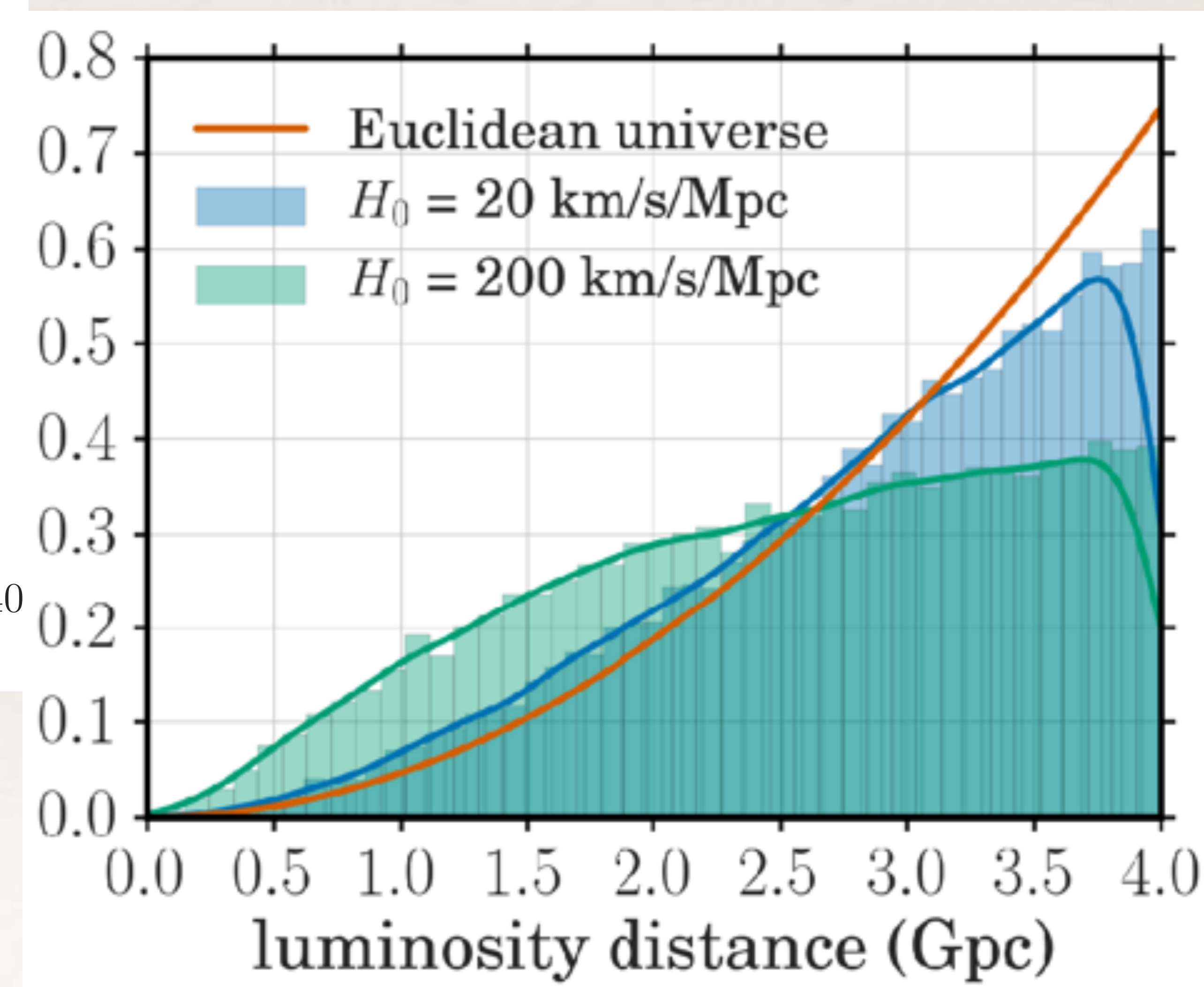
$$\times \left[\prod_{l=1}^{N_{\mathrm{obs}}} p(\mathbf{d}_l^{EM,\mathrm{obs}}|z_l, \vec{\Omega}_l, \vec{\lambda}_l) \prod_{m=N_{\mathrm{obs}}+1}^{N_{\mathrm{gal}}} p(\mathbf{d}_m^{EM,\mathrm{nobs}}|z_m, \vec{\Omega}_m, \vec{\lambda}_m) \right]$$

$$\times \left[\prod_{n=1}^{N_{\mathrm{gal}}} p(z_n, \vec{\Omega}_n) p(\vec{\lambda}_n|z_n, \vec{\Omega}_n) \right] p(N_{\mathrm{gal}})$$

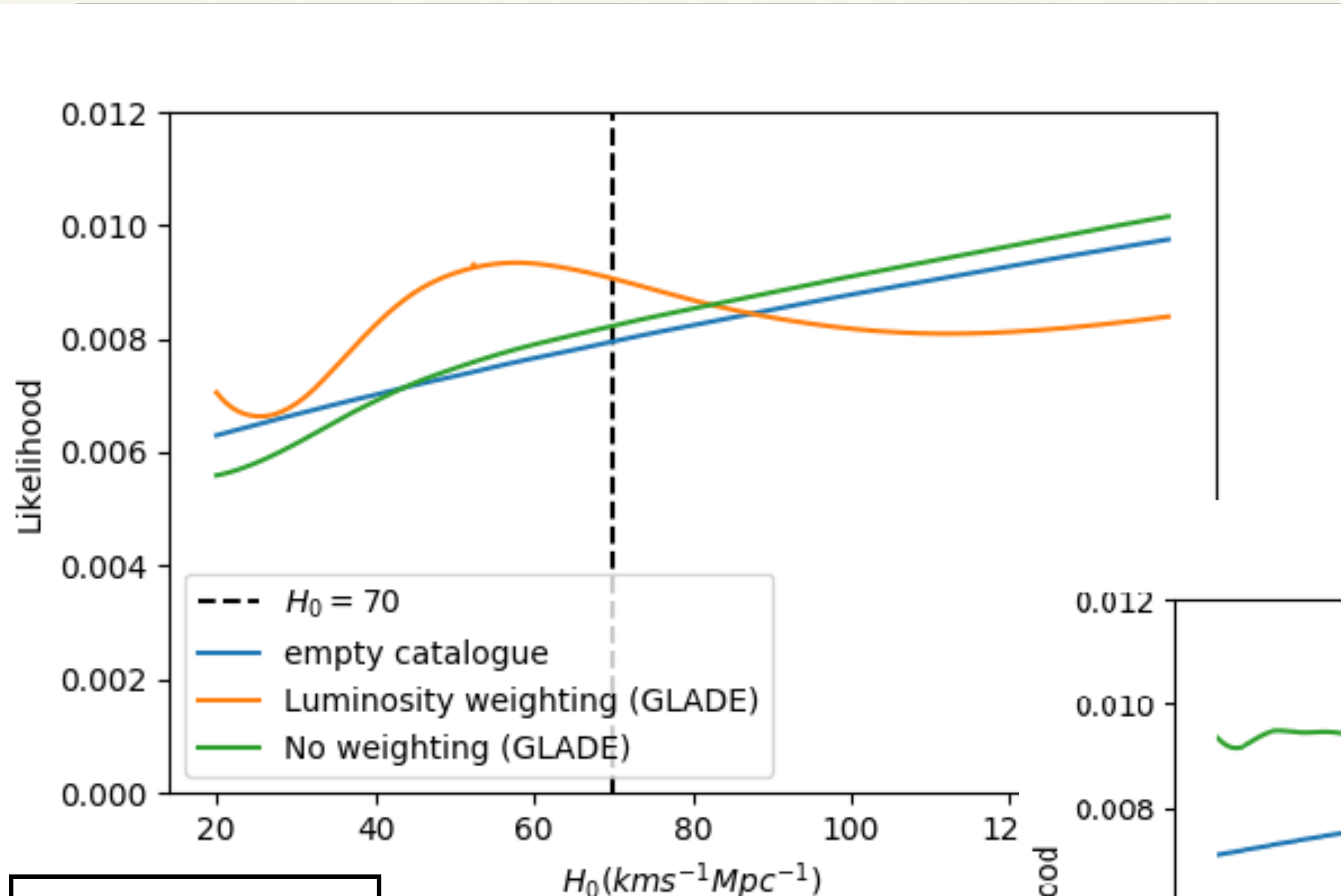
Sources of systematic error: population model

- Population uncertainties



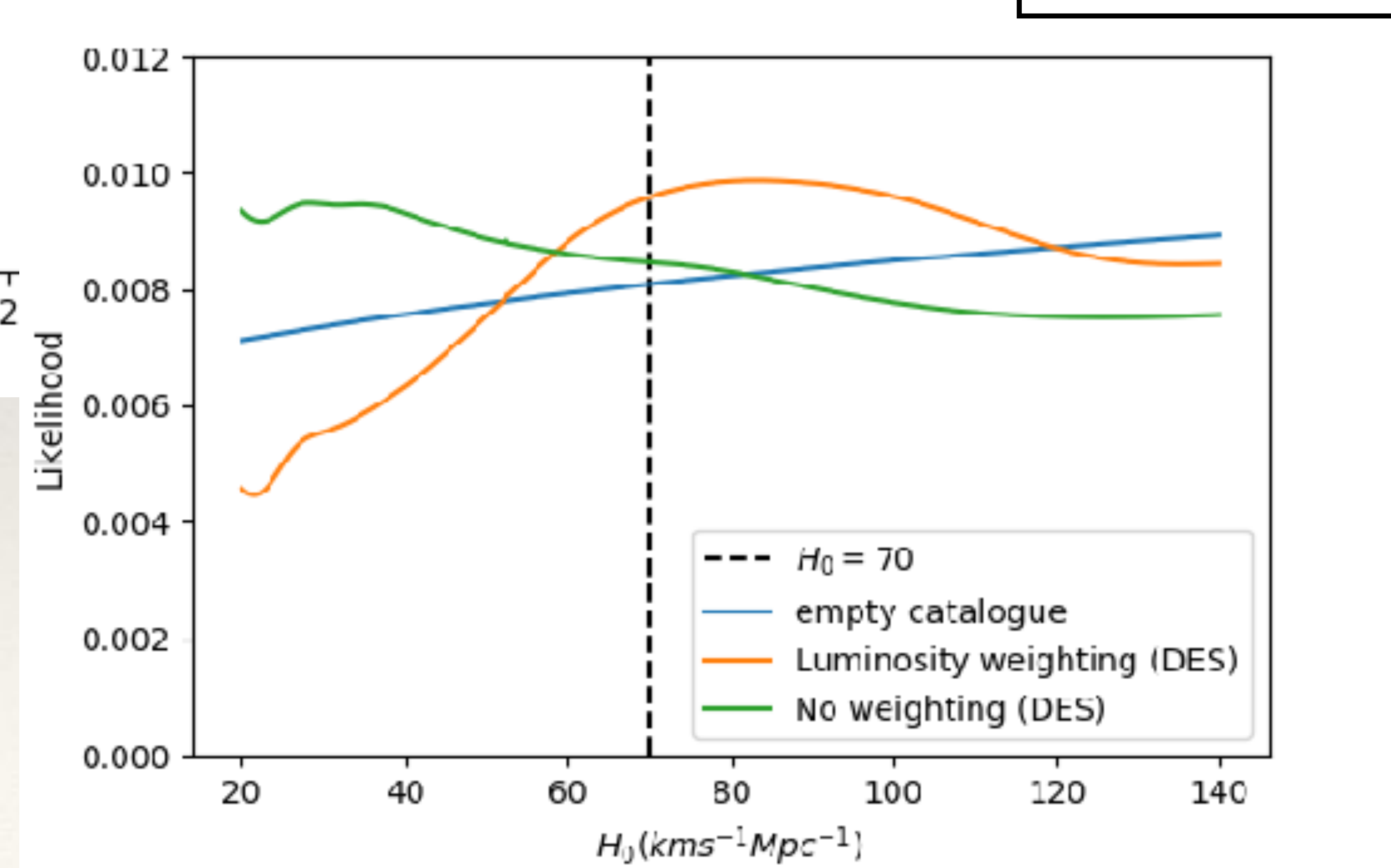


Sources of systematic error: host weights



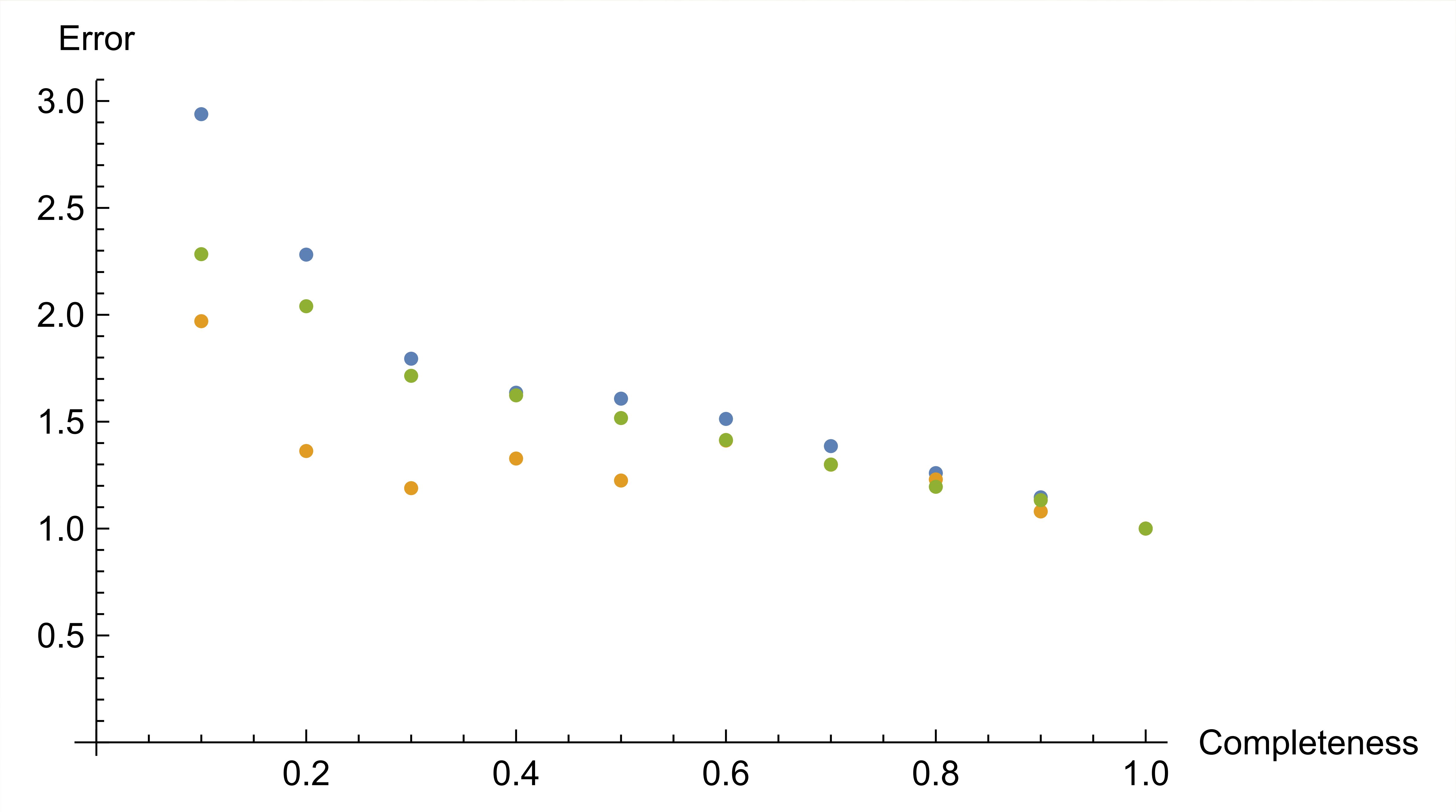
GW150914

GW170814



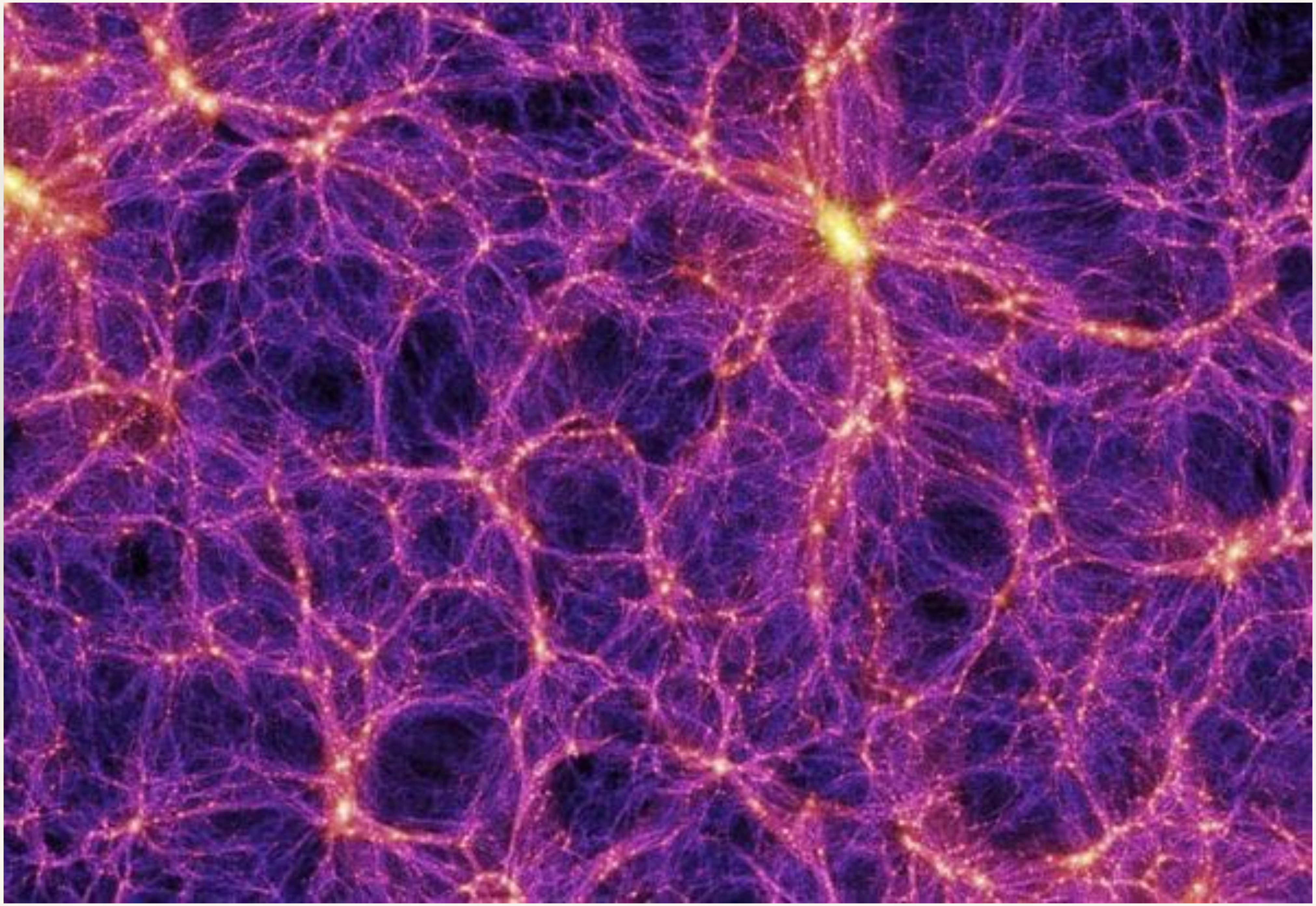
Sources of systematic error: completeness

- Catalogue incompleteness



Sources of systematic error: clustering

- Galaxy redshift distribution



Other sources of systematic error

- Impact of completeness motivates dedicated follow-up to improve catalogues. Galaxy clustering ensure it is the *effective* completeness rather than absolute completeness that matters, but must be modelled correctly.
- Other potentially important factors:
  - systematics in peculiar velocity corrections;
  - GW calibration errors;
  - modelling of GW and EM selection effects;
  - waveform model errors;
  - cosmological parameter uncertainties;
  - ……
- Can mitigate these uncertainties in the analysis, but must be aware of them, and mitigation will reduce precision of cosmological measurements.

Summary

- Gravitational waves are beginning to be used as standard sirens to probe the expansion of the Universe
  - First measurement of H_0 made possible by GW170817, since host galaxy redshift determined. Measured $H_0=70^{+12}_{-8}$ km s^{-1} Mpc^{-1}. With ~60-130 events, we will resolve current tension. After ~250-500 events we will obtain a 1% measurement.
  - Can also use statistical technique that doesn't require counterparts, or an assumed mass function. This method is less sensitive but can be applied to any GW event.
- GW measurements are not free from systematics, but these can be handled and are different to those from EM observations - they will provide a complementary probe with comparable sensitivity.
- This research programme brings together two topics to which Eddington made important early contributions!