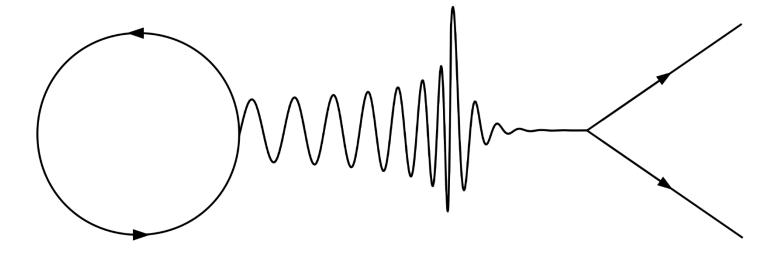
Black holes, atoms and gravitational molecules



Vítor Cardoso

(Técnico & CERN)

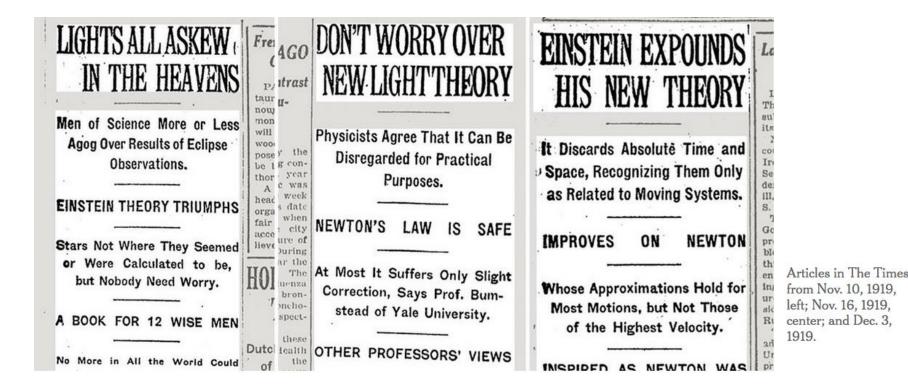
Image: Cardoso & Pani, CERN Courier (2016)











Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel and Costa 2012): A stationary, asymptotically flat, vacuum BH solution must be Kerr

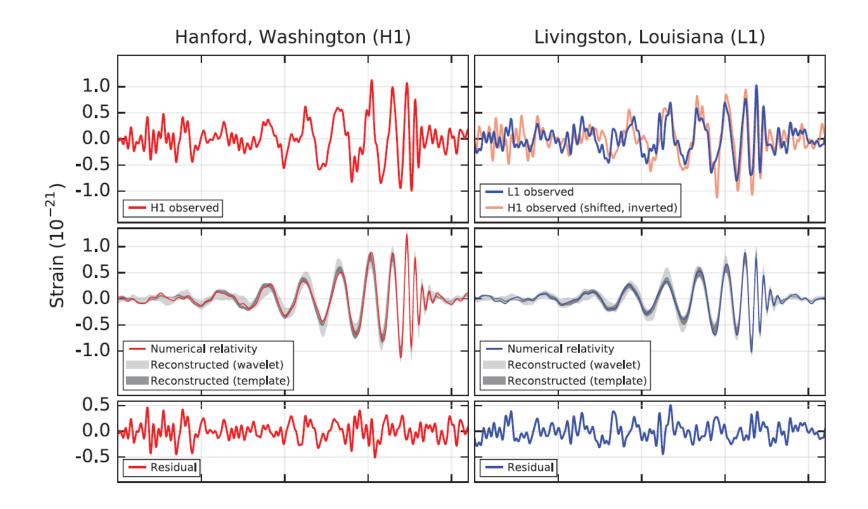
$$ds^{2} = \frac{\Delta - a^{2} \sin^{2} \theta}{\Sigma} dt^{2} + \frac{2a(r^{2} + a^{2} - \Delta) \sin^{2} \theta}{\Sigma} dt d\phi$$
$$- \frac{(r^{2} + a^{2})^{2} - \Delta a^{2} \sin^{2} \theta}{\Sigma} \sin^{2} \theta d\phi^{2} - \frac{\Sigma}{\Delta} dr^{2} - \Sigma d\theta^{2}$$
$$\Sigma = r^{2} + a^{2} \cos^{2} \theta, \quad \Delta = r^{2} + a^{2} - 2Mr$$

Describes a rotating BH with mass M and angular momentum J=aM, iff a<M

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe."

S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

They are out there, in isolation and in pairs



Abbott + Phys.Rev.Lett.116:061102 (2016)

Fundamental questions

a. BH seeds, BH demography, galaxy co-evolution (how many, where, how?) *See review Barack+ arXiv:1806.05195*

b. What is graviton mass or speed?

See review Barack+ arXiv:1806.05195

c. Are there extra radiation channels, corrections to gravity? *Barack+arXiv:1806.05195; Barausse+PRL116:241104(2016);*

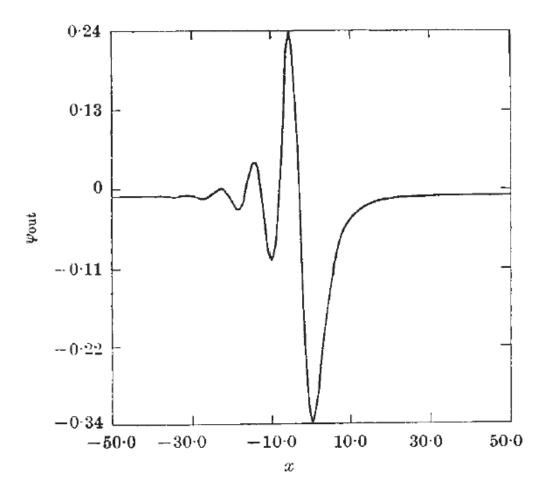
d. Is cosmic censorship preserved? Sperhake+ PRL103:131102 (2009); Cardoso+ PRL120:031103 (2018)

e. Is the final - or initial - object really a black hole? Cardoso+ PRL116: 171101 (2016); Cardoso & Pani, Nature Astronomy 1: 586 (2017)

f. Is it a Kerr black hole? Can we constrain alternatives? Berti+ 2005, 2016; Cardoso & Gualtieri 2016

g. Can GWs from BHs inform us on fundamental fields/DM? Barack+arXiv:1806.05195; Arvanitaki+ PRD95: 043001 (2016); Brito+ PRL119:131101 (2017)

Hair loss: the characteristic modes of black holes

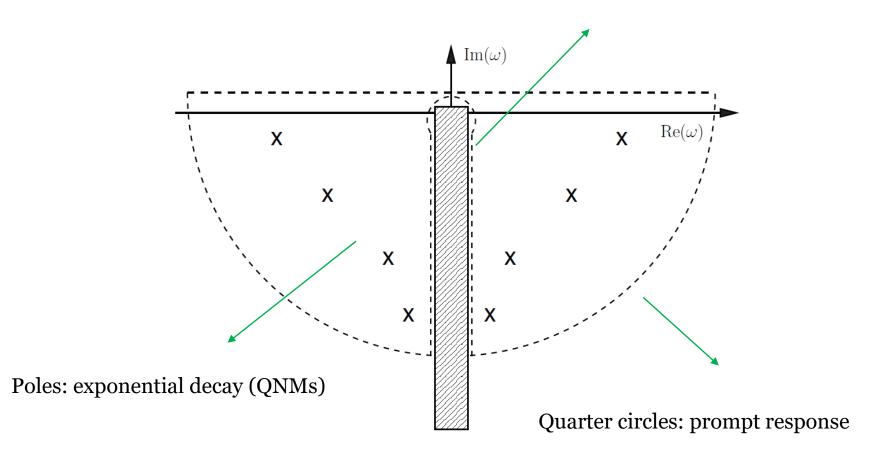


C.V.Vishveshwara, Nature 227: 938 (1970) Data and routines at blackholes.ist.utl.pt

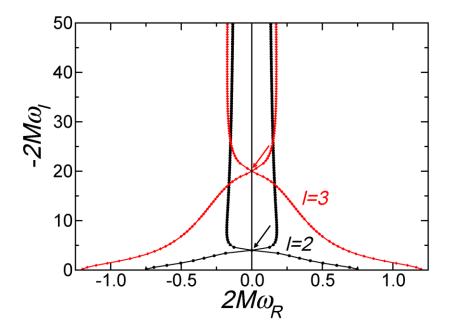
After harmonic decomposition

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}r_*^2} + (\omega^2 - V)\Psi = I(\omega, r)$$

Branch cut: power-law tails



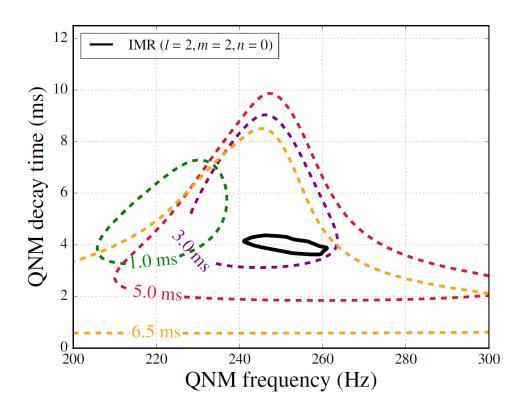
Leaver PRD34 1986



$$f = \omega_R / 2\pi = 1.207 \left(\frac{10 \ M_{\odot}}{M}\right) \text{kHz}$$
$$\tau = 1/|\omega_I| = 0.5537 \left(\frac{M}{10 \ M_{\odot}}\right) \text{ms}$$

Berti, Cardoso and Will PRD73: 064030 (2006) Berti, Cardoso and Starinets, CQG 26: 163001 (2009)

One and two-mode estimates



90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant

LIGO Collaboration PRL116:221101 (2016); Isi+ arXiv:1905.00869

Challenge: Can we "hear" the Kerr nature?

Need to measure two or modes: disentangle frequencies, damping times and amplitudes

$$\rho_{\rm GLRT}^{l=2,3} = 17.687 + \frac{15.4597}{q-1} - \frac{1.65242}{q}$$
$$\rho_{\rm GLRT}^{l=2,4} = 37.9181 + \frac{83.5778}{q} + \frac{44.1125}{q^2} + \frac{50.1316}{q^3}$$

Berti + PRD76: 104044 (2007) Berti + PRL117: 101102 (2016)

Challenge: can we estimate extra couplings?

Example: BH charge

(mini-charged DM predict heavy, fractional "electrons" and RN geometry: Rujula, Glashow, Sarid 1990; Perl, Lee 1997; Holdom 1986; Sigurdson + 2004)

$$\mathcal{L} = \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + 4\pi e j_{\rm em}^{\mu} A_{\mu} + 4\pi e_h j_h^{\mu} B_{\mu} + 4\pi \epsilon e j_h^{\mu} A_{\mu} \right)$$

Ringdown bound:

$$\frac{Q}{M} \lesssim 0.1 \sqrt{\frac{100}{\rho}}$$

And can be generalized to other theories, provided spectra is known

Cardoso + JCAP 1605: 054 (2016) Blázquez-Salcedo + PRD94:104024 (2016)

The evidence for black holes

Cardoso and Pani, Living Reviews in Relativity arXiv:1904.05363

	Constraints		Source	
	$\epsilon(\lesssim)$	$\frac{\nu}{\nu_{\infty}} \gtrsim$		
1a. 1b.	$\begin{array}{c} \mathcal{O}(1) \\ 0.74 \end{array}$	$\mathcal{O}(1) \\ 1.5$	Sgr A* & M87 GW150914	ISCO and Light ring Merger frequency
2.	$\mathcal{O}(0.01)$	$\mathcal{O}(10)$	GW150914	Ringdown consistency
3.	$10^{-4.4}$	158	All with $M > 10^{7.5} M_{\odot}$	Lack of optical/UV in TDR
4.	10^{-14}	10^{7}	Sgr A^*	Low relative luminosity of SgrA
5.	10^{-40}	10^{20}	All with $M < 100 M_{\odot}$	No ergoregion stochastic
6.	10^{-47}	10^{23}	GW150914	Absence of echoes
7*.	$e^{-10^4/\zeta}$	$e^{5000/\zeta}$	EMRIs	Projected constraints on spin- induced quadrupole and TLNs

Plenty of caveats, but enormous potential

GWs and dark matter I

Dark matter is not a strong-field phenomenon, but GW observations may reveal a more "mundane" explanation in terms of heavy BHs

Bird + PRL116:201301 (2016)

Π

Inspiral occurs in dark-matter rich environment and may modify the way inspiral proceeds, given dense-enough media: accretion and gravitational drag play important role.

Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013)

DM II

Inspiral occurs in dark-matter rich environment and may modify the way inspiral proceeds, given dense-enough media: accretion and gravitational drag play important role.

Eda + *PRL*110:221101 (2013); *Macedo* + *ApJ*774:48 (2013); *Barausse*+*PRD* 2014

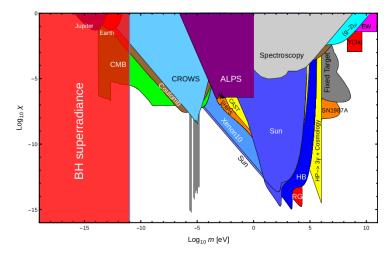
Self-gravity:

$$\rho_0 = 10^3 M_{\odot} \text{pc}^{-3} \sim 10^4 \text{GeV cm}^{-3}$$
$$\frac{M_{\text{inside r}}^{\text{DM}}}{M_{\text{BH}}} = 10^{-19} \left(\frac{M_{\text{BH}}}{10^6 M_{\odot}}\right)^2 \left(\frac{r}{100M}\right)^3 \frac{\rho_{\text{DM}}}{\rho_0}$$

Accretion:

$$\dot{M}_{\rm BH} = \frac{16\pi G^2 M_{\rm BH}^2 \rho_{\rm DM}}{v_{\rm DM} c^2} \left(\dot{M} = \sigma \rho v \right)$$
$$\frac{\Delta M_{\rm BH}}{M_{\rm BH}} = 10^{-16} \left(\frac{M_{\rm BH}}{10^6 M_{\odot}} \right) \frac{\rho_{\rm DM}}{\rho_0} \frac{T}{1 \, \text{year}} \left(\frac{\sigma_v}{220 \, \text{Km/s}} \right)^{-1}$$

DM III. Light fields



Cardoso+ 2018, adapted from Sigl (2017) and Jaeckel arXiv:1303.1821

Interesting as effective description; proxy for more complex interactions; arise as interesting extensions of GR^* (*BD or generic ST theories, f(R), etc)*

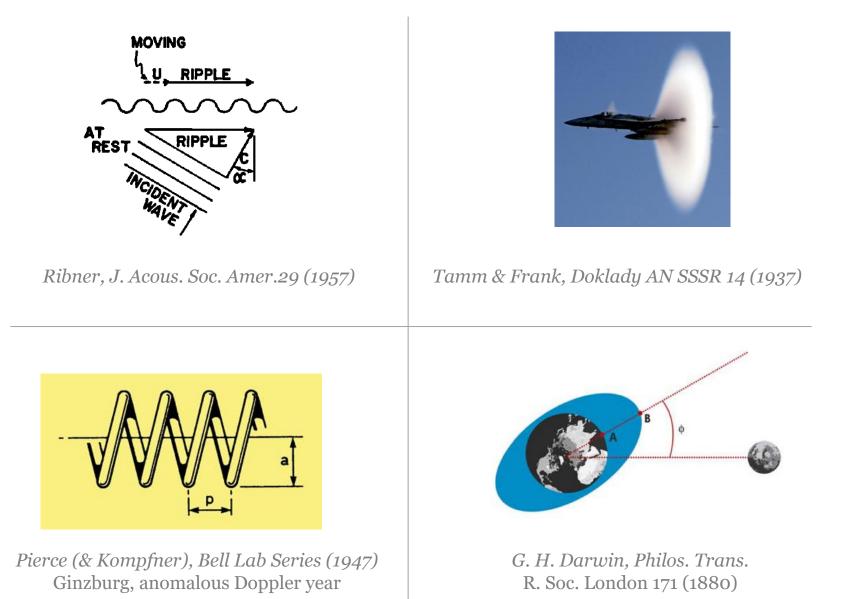
Bosons do exist (Higgs) and lighter versions may as well Peccei-Quinn (interesting because not invented to solve DM problem), axiverse (moduli and coupling constants in string theory)

$$\mathcal{L} = \frac{R}{k} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \Psi \partial_{\nu} \Psi - \frac{\mu_{\rm S}^2}{2} \Psi \Psi - \frac{k_{\rm axion}}{2} \Psi * F^{\mu\nu} F_{\mu\nu}$$

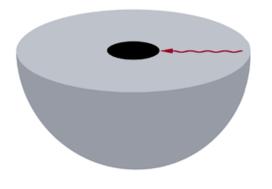
...and one or more could be a component of DM. D. Marsh, Phys. Repts. 2016

Superradiance

Zel'dovich JETP Lett. 14:180 (1971); Brito+ Lect. Notes Phys.906 (2015)



Bombs and superradiant instabilities



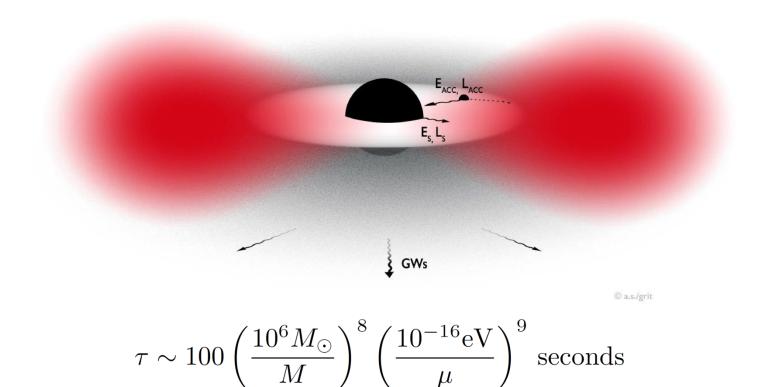
B A.S./Dy8Ho

$$\tau \sim 100 \left(\frac{10^6 M_{\odot}}{M}\right)^8 \left(\frac{10^{-16} \text{eV}}{\mu}\right)^9 \text{ seconds}$$

Massive "states" around Kerr are linearly unstable

See review Brito, Cardoso, Pani, Lect. Notes Phys. 906: 1 (2015); arXiv:1501.06570

Fundamental fields: bounding the boson mass

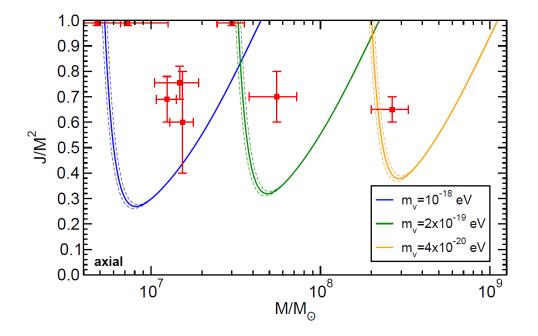


Wonderful sources of GWs

Brito, Cardoso, Pani, Lecture Notes Physics 906: 1-237 (2015)

Bounding the boson mass with EM observations

Pani + PRL109, 131102 (2012)

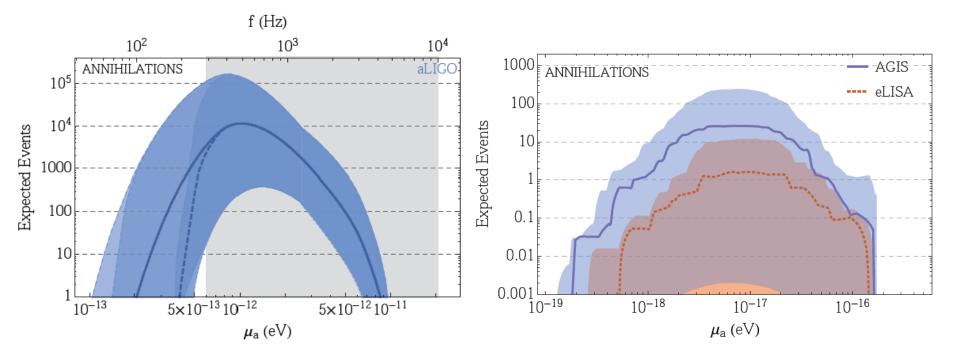


Bound on photon mass is model-dependent: details of accretion disks or intergalactic matter are important... but gravitons interact very weakly!

$$m_g < 5 \times 10^{-23} \,\mathrm{eV}$$

Brito + PRD88:023514 (2013); Review of Particle Physics 2014

Wonderful sources for different GW-detectors!



Arvanitaki+ PRD91:084011 (2015);Brito+CQG32:134001 (2015); Brito+ Lect.Notes Physics 906 (2015)

Wonderful sources for different GW-detectors!

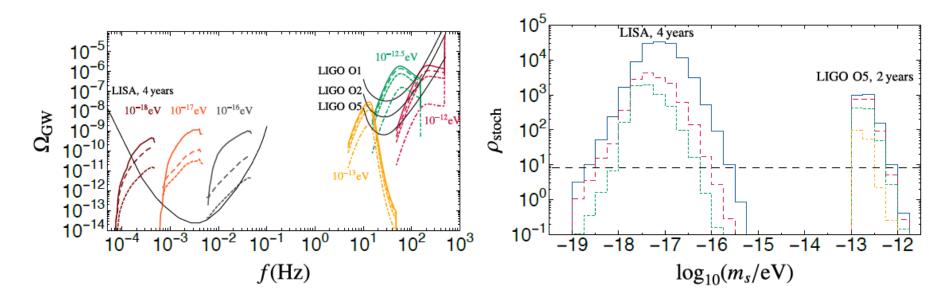


FIG. 2. Left panel: stochastic background in the LIGO and LISA bands. For LISA, the three different signals correspond to the "optimistic" (top), "less optimistic" (middle) and "pessimistic" (bottom) astrophysical models. For LIGO, the different spectra for each scalar field mass correspond to a uniform spin distribution with (from top to bottom) $\chi_i \in [0.8, 1], [0.5, 1], [0, 1]$ and [0, 0.5]. The black lines are the power-law integrated curves of Ref. [61], computed using noise PSDs for LISA [9], LIGO's first two observing runs (O1 and O2), and LIGO at design sensitivity (O5) [62]. By definition, $\rho_{\text{stoch}} \ge 1$ when a power-law spectrum intersects one of the power-law integrated curves. Right panel: ρ_{stoch} for the backgrounds shown in the left panel. We assumed $T_{\text{obs}} = 2$ yr for LIGO and $T_{\text{obs}} = 4$ yr for LISA.

Brito + PRL119: 131101 (2017); arXiv 1706:05097

Couplings and EM bursts

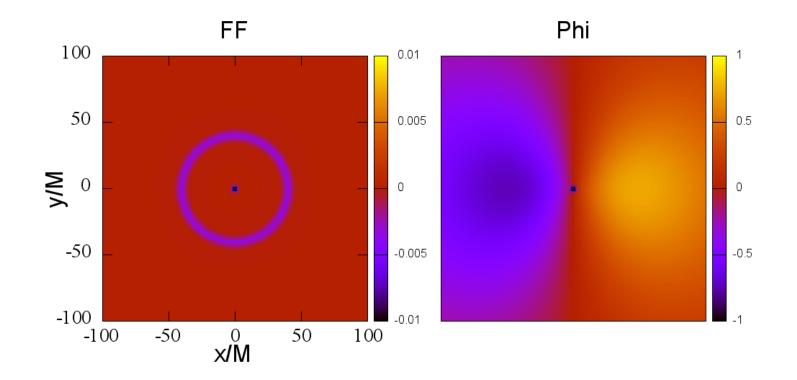
Consider couplings to SM

$$\begin{split} \mathcal{L} &= \frac{R}{k} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \Psi \partial_{\nu} \Psi - \frac{\mu_{\rm S}^2}{2} \Psi \Psi - \frac{k_{\rm axion}}{2} \Psi \ ^* F^{\mu\nu} F_{\mu\nu} \\ & \Psi \sim \Psi_0 e^{-i\mu_S t} , \qquad A_\mu \sim y(t) e^{ip_\mu x^\mu} \\ & \frac{\partial^2 y}{\partial t^2} + \left(\omega^2 - 2\mu_S \Psi_0 k_{\rm axion} |p| \cos \mu_S t\right) y = 0 \\ & \text{Blasts of EM, laser-like radiation for} \end{split}$$

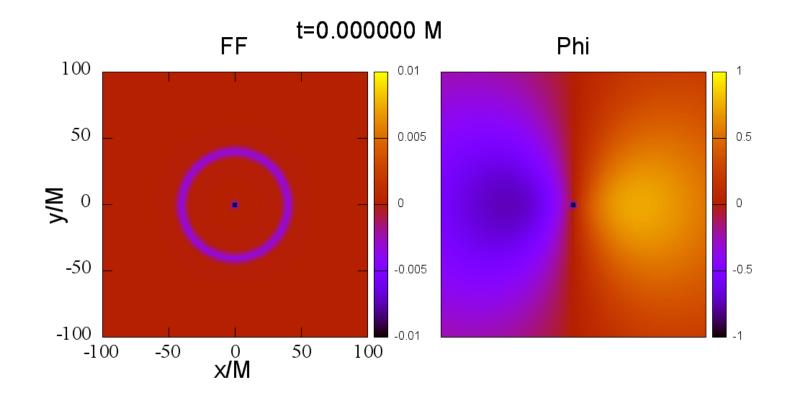
 μ >10⁻⁸eV, M<0.01 M_{sun}

Boskovic+ PRD99:035006 (2019); Ikeda+ PRL122:081101 (2019)

t=0.000000 M



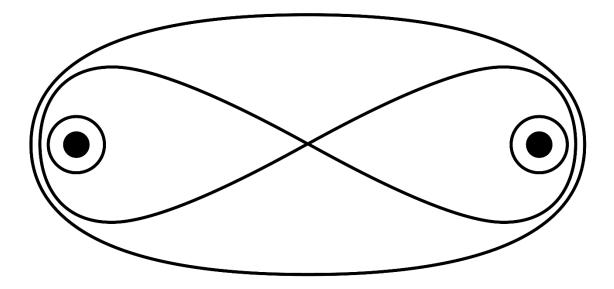
Ikeda, Brito and Cardoso PRL122: 081101 (2019)



Ikeda, Brito and Cardoso PRL122: 081101 (2019)

Gravitational molecules: a toy model

$$ds^{2} = -\frac{dt^{2}}{U^{2}} + U^{2} \left(d\rho^{2} + \rho^{2} d\phi^{2} + dz^{2} \right)$$
$$U(\rho, z) = 1 + \frac{M}{\sqrt{\rho^{2} + (z - a)^{2}}} + \frac{M}{\sqrt{\rho^{2} + (z + a)^{2}}}$$



Chandrasekhar PRSLA421:227 (1989); Assumpção+ PRD98: 064036(2018)

Gravitational molecules: a toy model

Change to prolate confocal elliptic coordinates

$$\rho^{2} + (a - z)^{2} = a^{2}(\chi + \eta)^{2}$$
$$\rho^{2} + (a + z)^{2} = a^{2}(\chi - \eta)^{2}$$

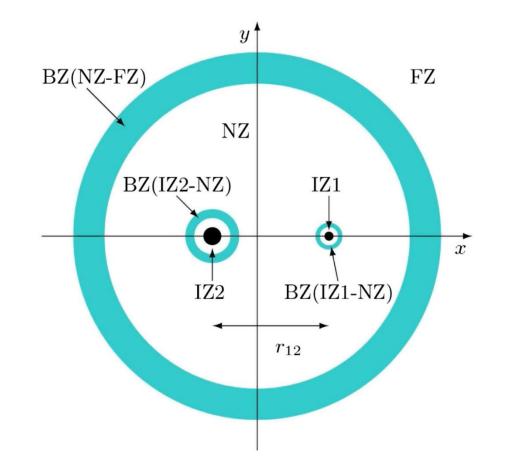
$$\partial_{\eta} \left((1 - \eta^2) \partial_{\eta} S \right) + \left(-a^2 \omega^2 \eta^2 - \frac{m^2}{1 - \eta^2} + \Lambda \right) S = 0$$

$$\partial_{\chi} \left((\chi^2 - 1) \partial_{\chi} R \right) + \left(a^2 \omega^2 \chi^2 + 8Ma\chi \, \omega^2 - \frac{m^2}{\chi^2 - 1} - \Lambda \right) R = 0$$

Klein-Gordon equation is identical to Schrodinger for Di-Hydrogen ionized molecule! Bernard+ (2019)

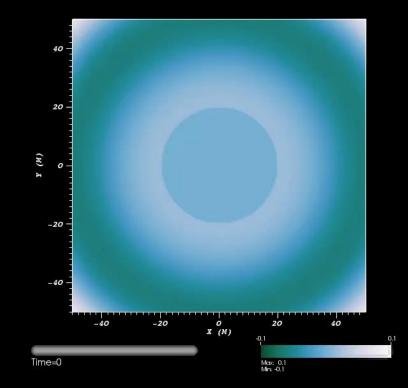
for Hydrogen molecule see Burrau M7: 1 (1928); Wilson PRSLA118:635 (1929); Hylleraas ZfP71: 739 (1931)

Gravitational molecules: a real BH binary



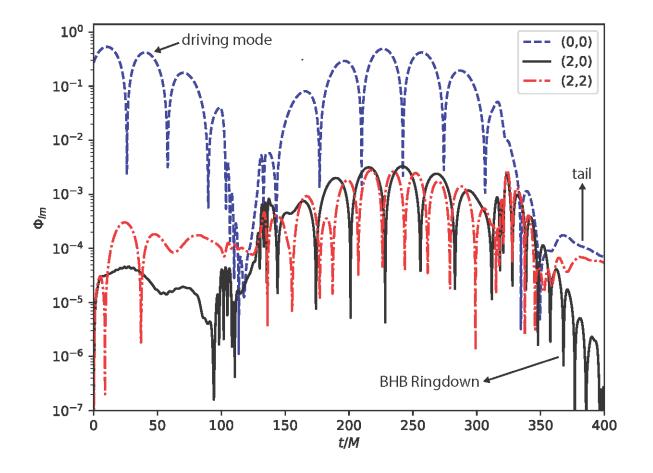
Mundim+ PRD89: 084008 (2014); Bernard + (2019)

Gravitational molecules: a real BH binary



Bernard + arXiv:1905.05204 (and work in progress)

Gravitational molecules: a real BH binary

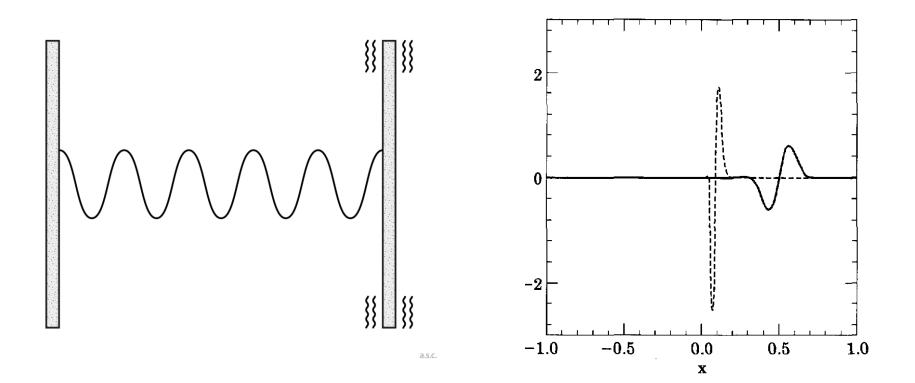


Global BHB modes may be resonantly excited

Bernard + arXiv:1905.05204 (and work in progress)

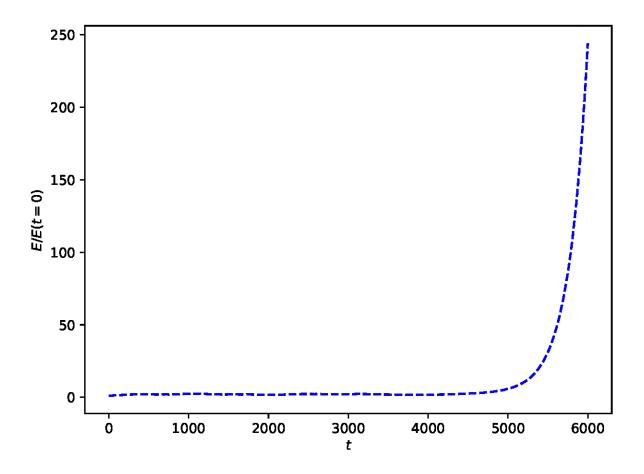
Energy extraction from black hole binaries?

- 1. Superradiance, if individual black holes spin
- 2. Ergoregions in binaries?
- 3. Slingshot effect for massless waves
- 4. Parametric resonance? Fermi-like acceleration?



Cooper IEEE Trans. Ant. Propag. 1993

Energy extraction from binaries?



Bernard + arXiv:1905.05204 (and work in progress)

Conclusions: exciting times!

Gravitational wave astronomy *can* become a precision discipline, mapping compact objects throughout the entire visible universe.

Black holes remain the most outstanding object in the universe. Simple as Hydrogen atoms, BH spectroscopy will allow to test GR and possibly the presence of horizons. BHs cluster in binaries, which - like molecules - have their own spectra.

"After the advent of gravitational wave astronomy, the observation of these resonant frequencies might finally provide direct evidence of BHs with the same certainty as, say, the 21 cm line identifies interstellar hydrogen"

(S. Detweiler ApJ 239:292 1980)

Thank you



Environment: ringdown properties

Barausse + PRD89:104059 (2014)

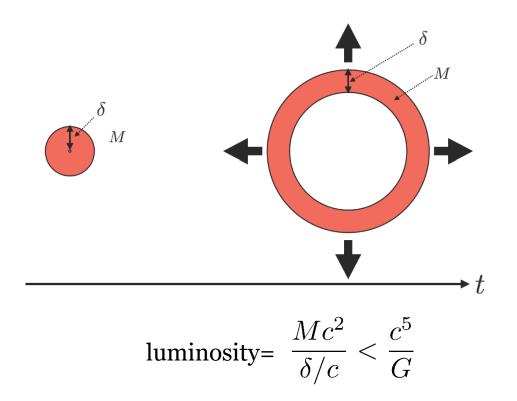
Correction	$ \delta_R [\%]$	$ \delta_I [\%]$
spherical near-horizon distribution	0.05	0.03
ring at ISCO	0.01	0.01
electric charge	10^{-5}	10^{-6}
magnetic field	10^{-8}	10^{-7}
gas accretion	10^{-11}	10^{-11}
DM halos	$10^{-21} \rho_3^{\rm DM}$	$10^{-21} \rho_3^{\rm DM}$
cosmological effects	10^{-32}	10^{-32}

On the maximum luminosity

Event	Peak Luminosity	
Three Gorges dam	$3 \times 10^{-43} \mathcal{L}_{\rm P}$	
Most powerful laser	$3 \times 10^{-38} \mathcal{L}_{\rm P}$	
Tsar Bomba	$3 \times 10^{-27} \mathcal{L}_{\rm P}$	
Solar luminosity	$1 \times 10^{-30} \mathcal{L}_{\rm P}$	
γ -ray bursts	$1 \times 10^{-5} \mathcal{L}_{\mathrm{P}}$	
Inspiralling BHs	$2 \times 10^{-3} L_{\rm P}$	
High-energy BH collision	$2 \times 10^{-2} \mathcal{L}_{\rm P}$	
Critical collapse	$2 \times 10^{-1} \mathcal{L}_{\mathrm{P}}$	
End point of BH evaporation	$\mathcal{L}_{ ext{P}}$	

$$\mathcal{L}_{\rm P} = \frac{c^5}{G} = 3.6 \times 10^{52} \,{\rm W}$$

The Thorne-Dyson conjecture



K. Thorne, Gravitational Radiation (North-Holland 1983) Gibbons & Barrow MNRAS 446:3874 (2015) Cardoso+ PRD97:084013 (2018)