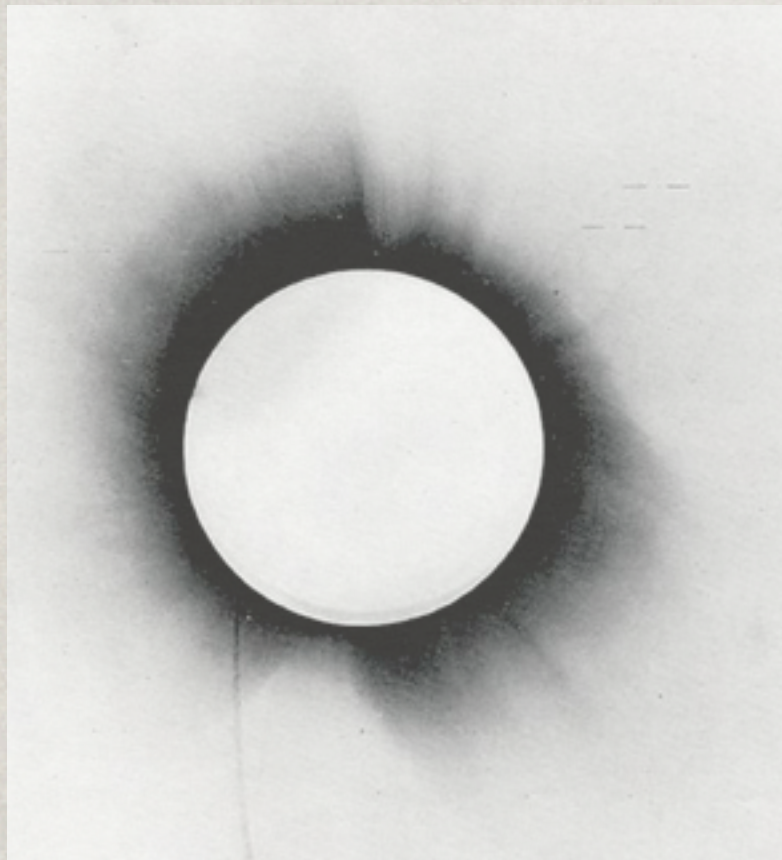


Black hole shadows and strong gravitational lensing



1919



2019

Carlos A. R. Herdeiro
IST-Lisbon Physics Dept. and CENTRA

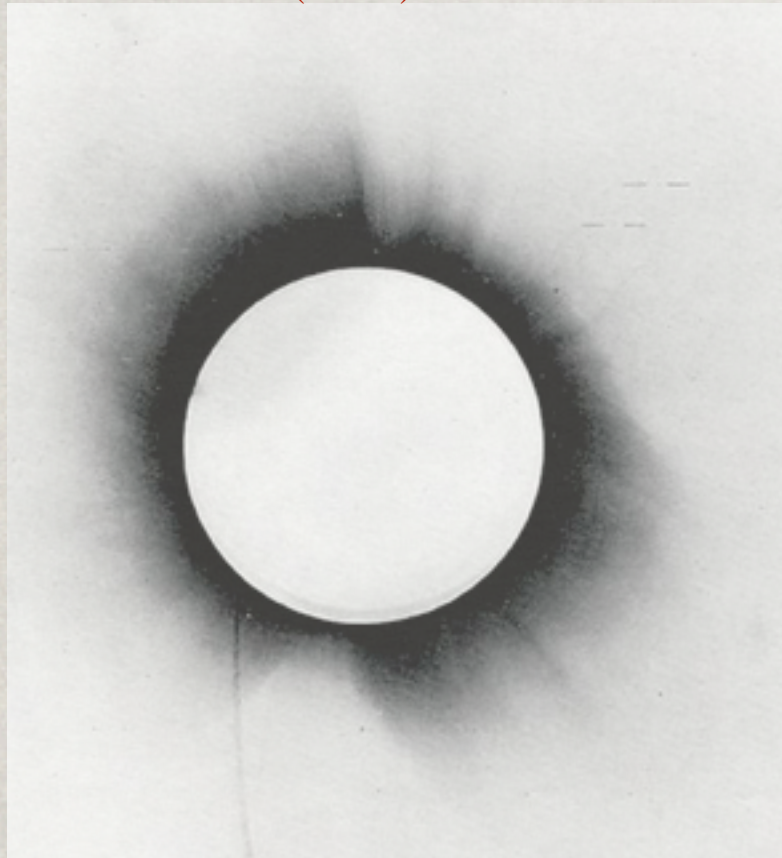
Based on work (mostly) with P. Cunha, E. Radu

Eddington at Sundry, Príncipe Island, São Tomé e Príncipe
May 27th 2019



100 years of (observational) gravitational lensing

Dyson, Eddington and Davidson
Phil. Trans. Royal Soc. London
220 (1920) 571-581



1919

EHT collaboration
ApJ Lett. 875 (2019) L1

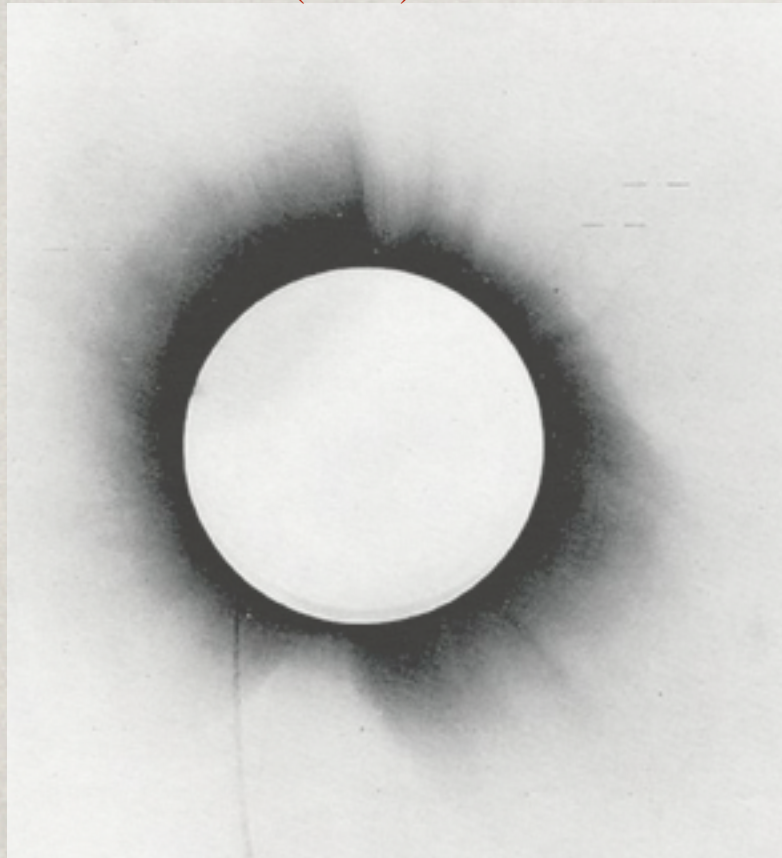


2019

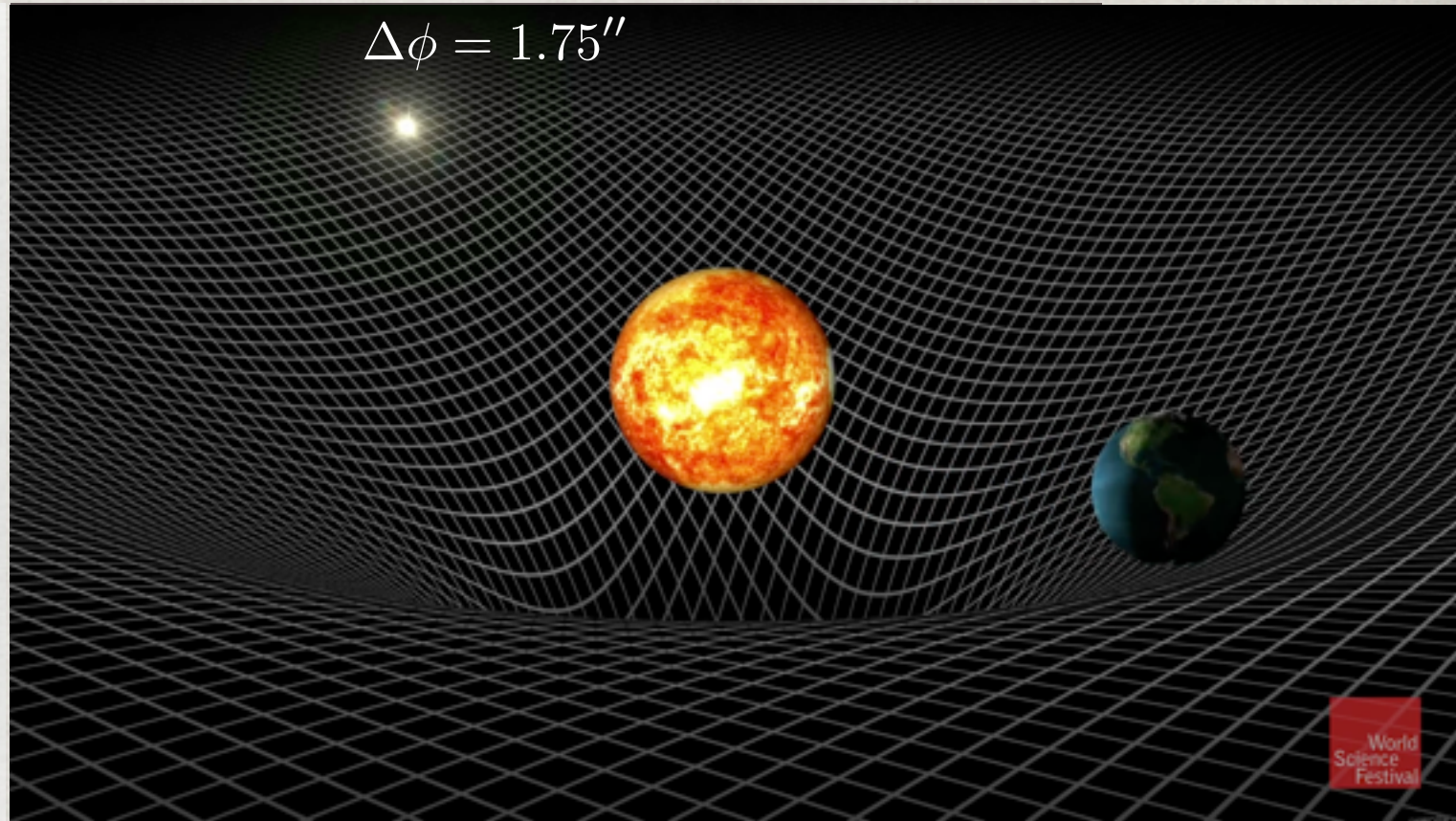
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key
to
establish
GR



1919



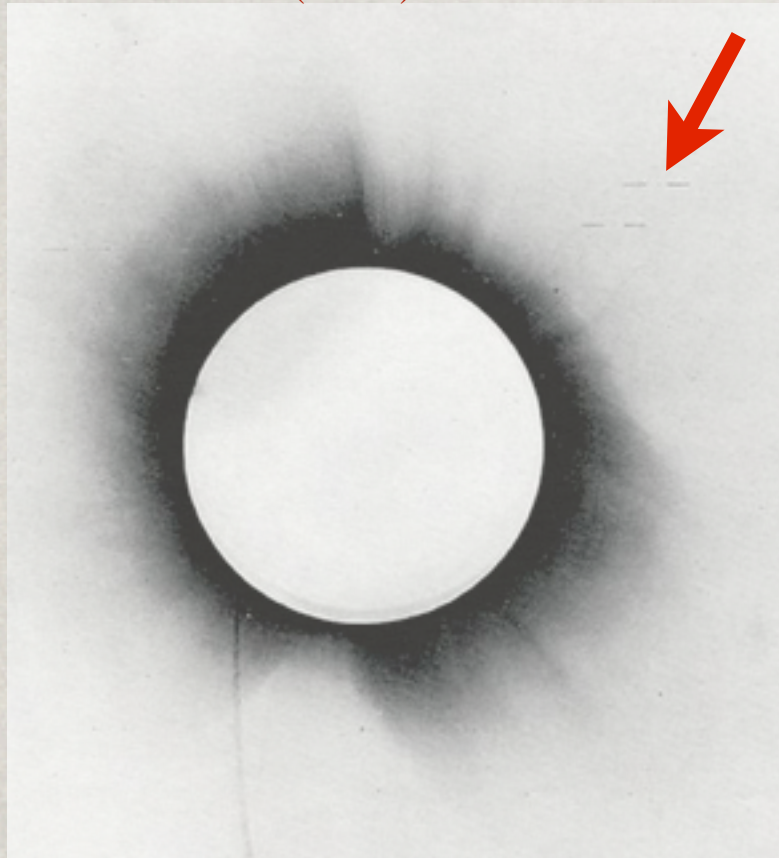
weak lensing

$$\Delta\phi = \frac{4GM}{c^2 d} = 1.75'' \frac{r_\odot}{d}$$

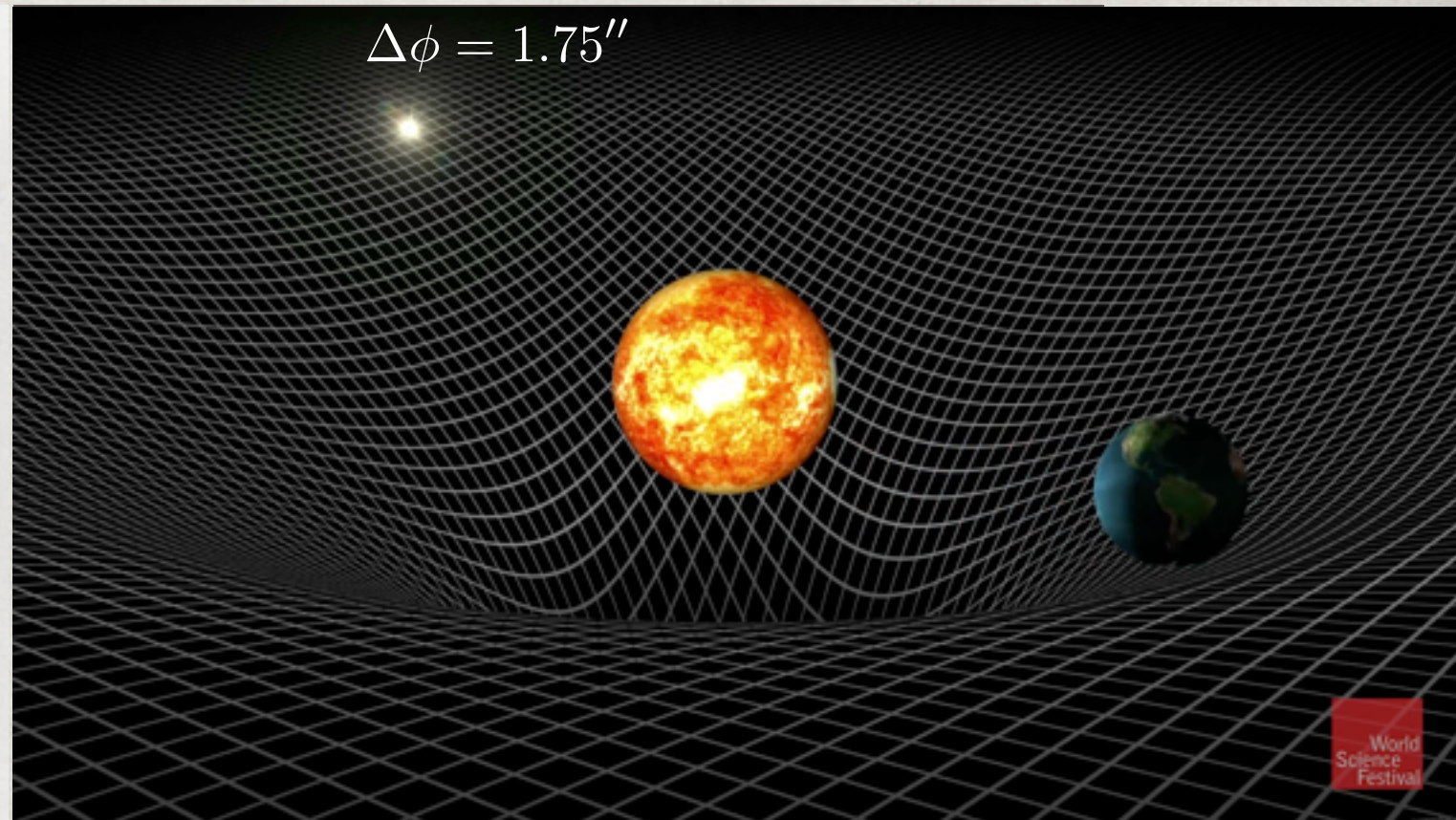
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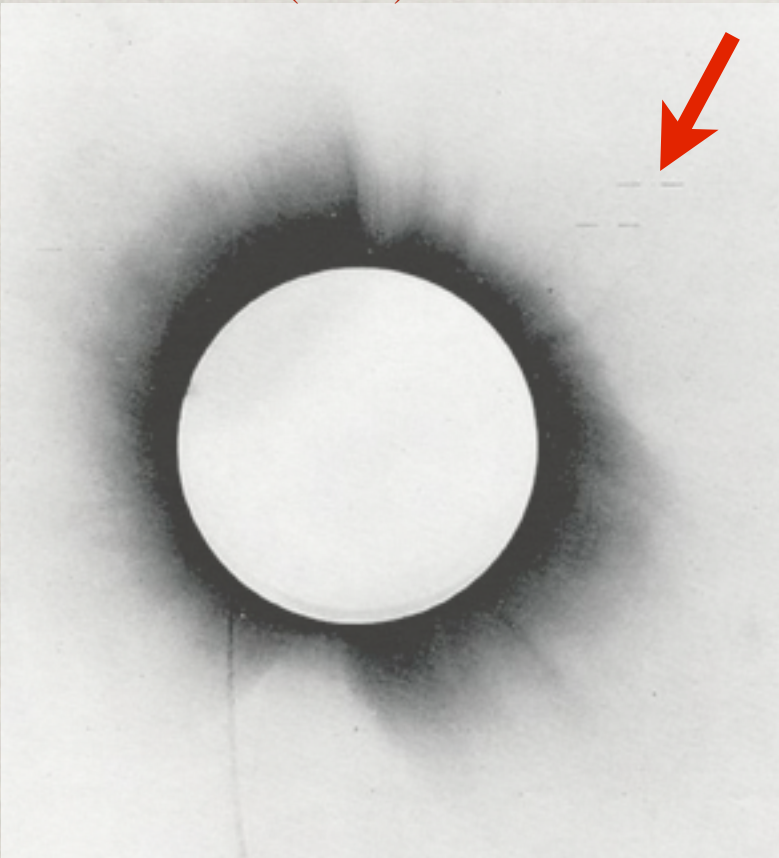


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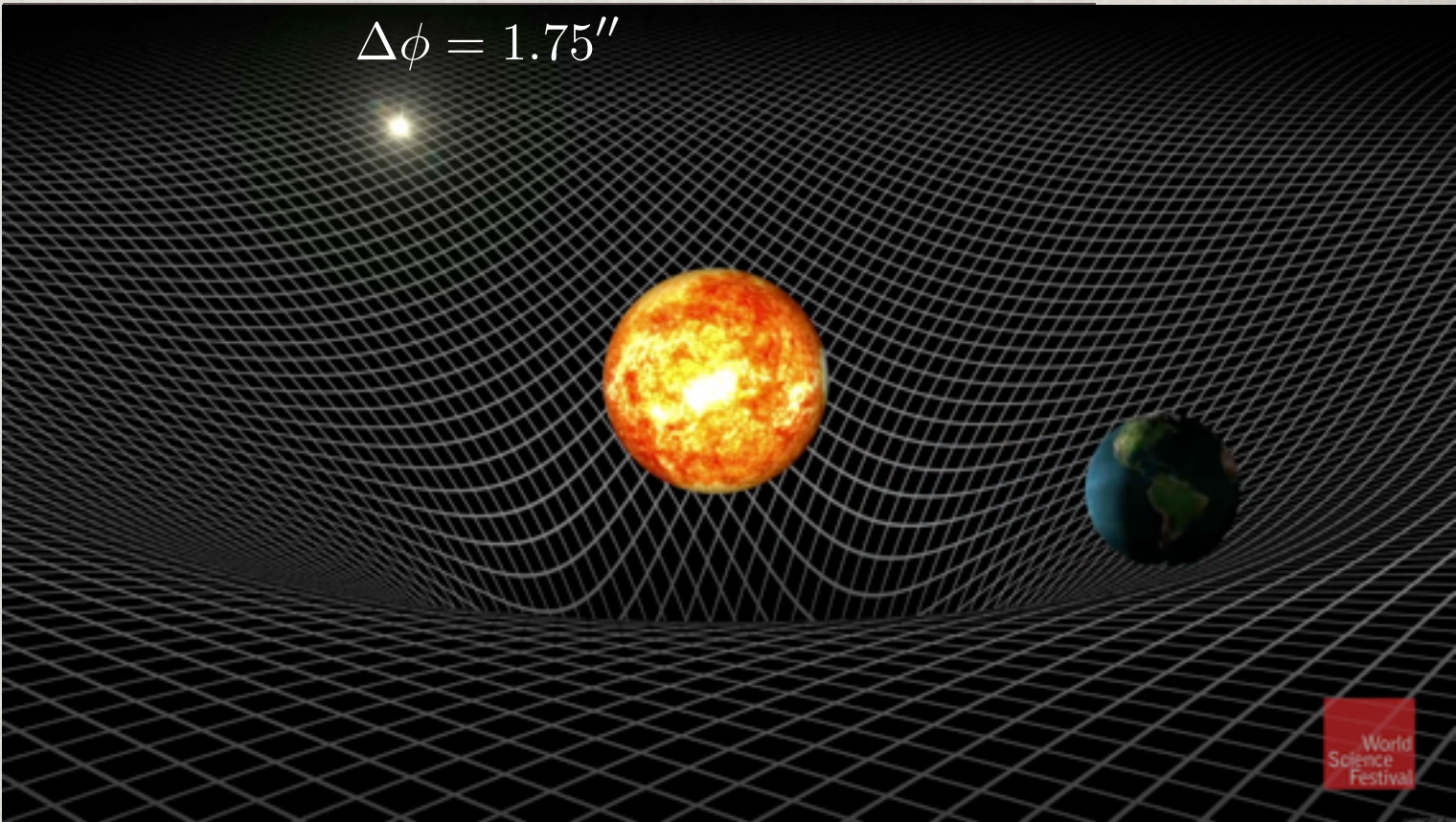


Table 1 | Experimental measurements (in arcsecond) obtained from the 1919 plates

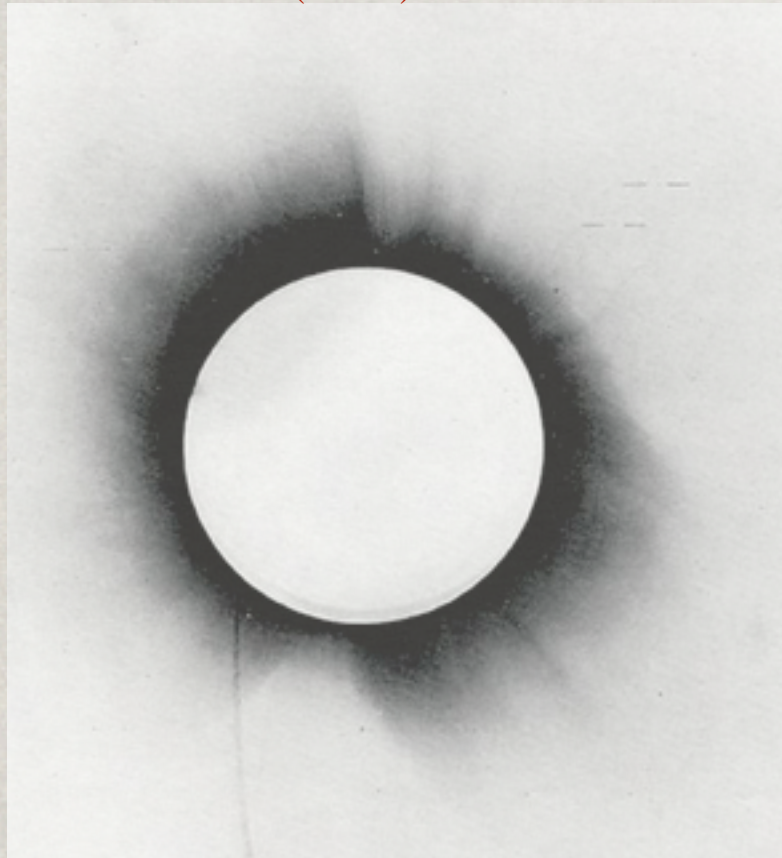
Instrument	1919 result ¹	1979 re-analysis ⁹
Príncipe astrographic	1.61 ± 0.30	–
Sobral 4 inch	1.98 ± 0.18	1.90 ± 0.11
Sobral astrographic	0.93 ± 0.50 or 1.52 ± 0.46	1.55 ± 0.34

The middle column shows the results obtained in 1919¹, including results from two different calculations based on the Sobral astrographic data, and the results obtained from the re-measurement of the Sobral plates, carried out later at the Royal Greenwich Observatory⁹, are displayed in the right column.

Crispino and Kennefick
Nature Physics, 15 (may 2019) 416

One century of gravitational lensing

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Phil. Trans. Royal Soc. London
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EHT collaboration
ApJ Lett. 875 (2019) L1

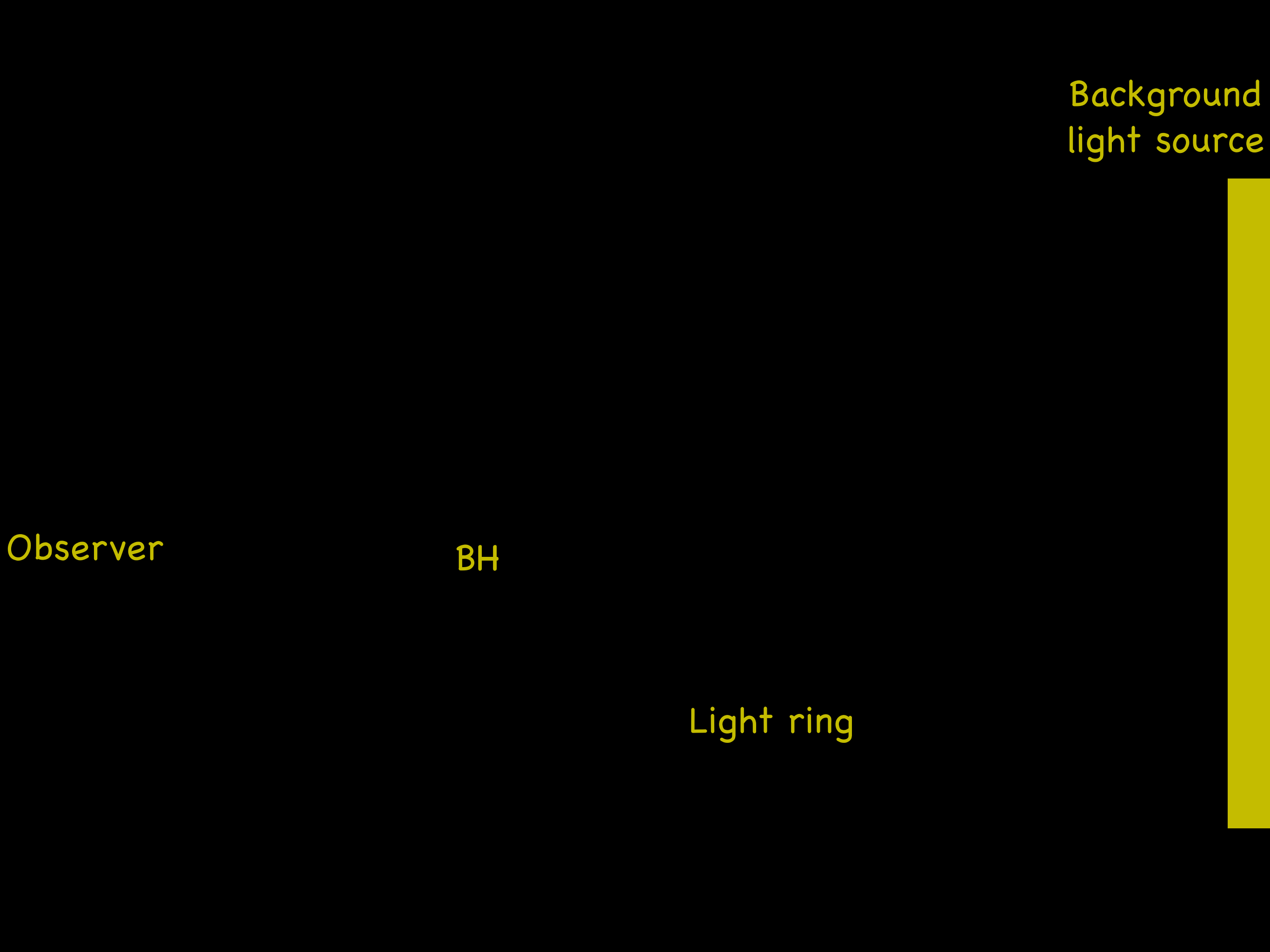


2019

strong lensing

$$\Delta\phi > 2\pi$$

Bound orbits of light:
light rings and fundamental photon orbits

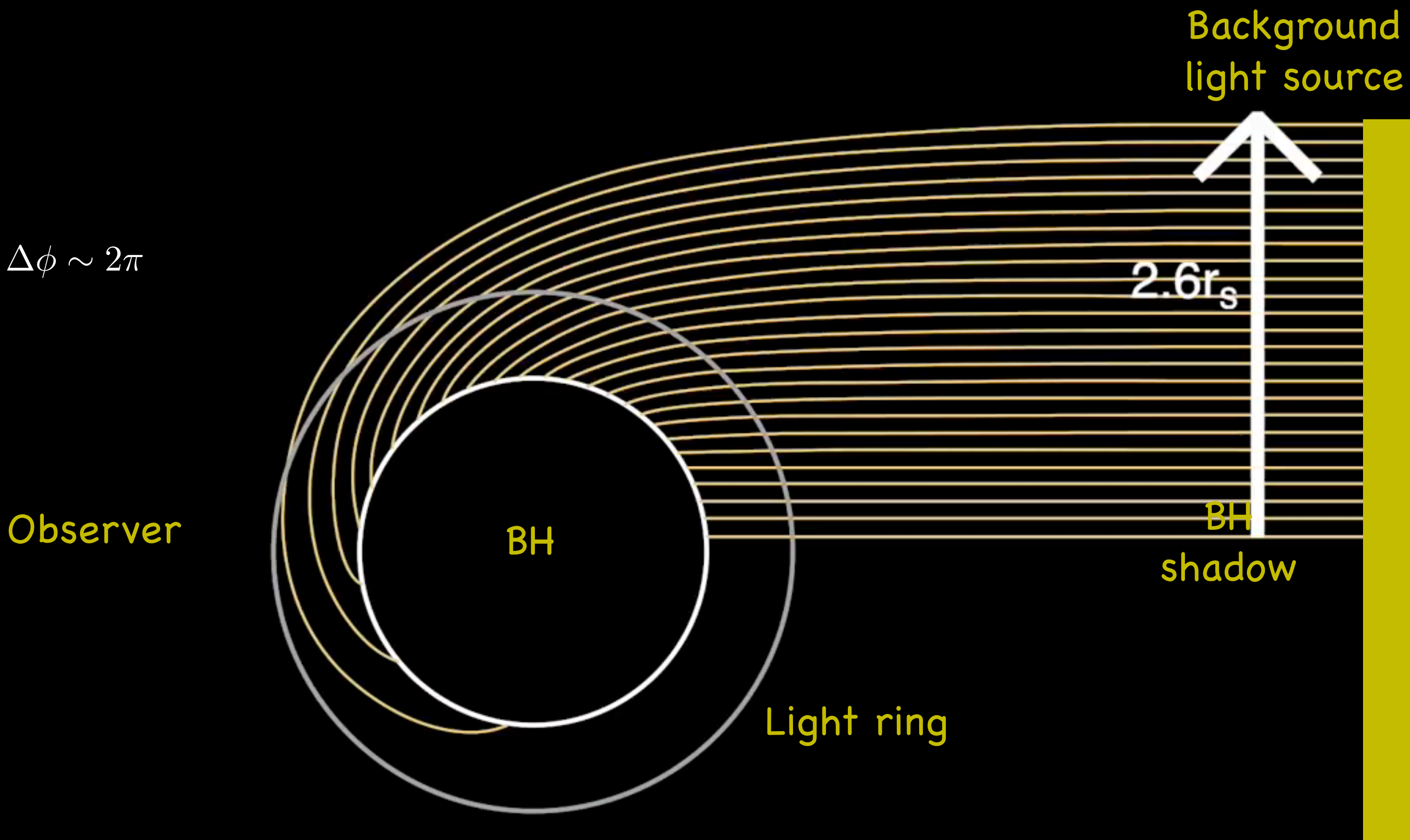


Background
light source

BH

Light ring

Observer

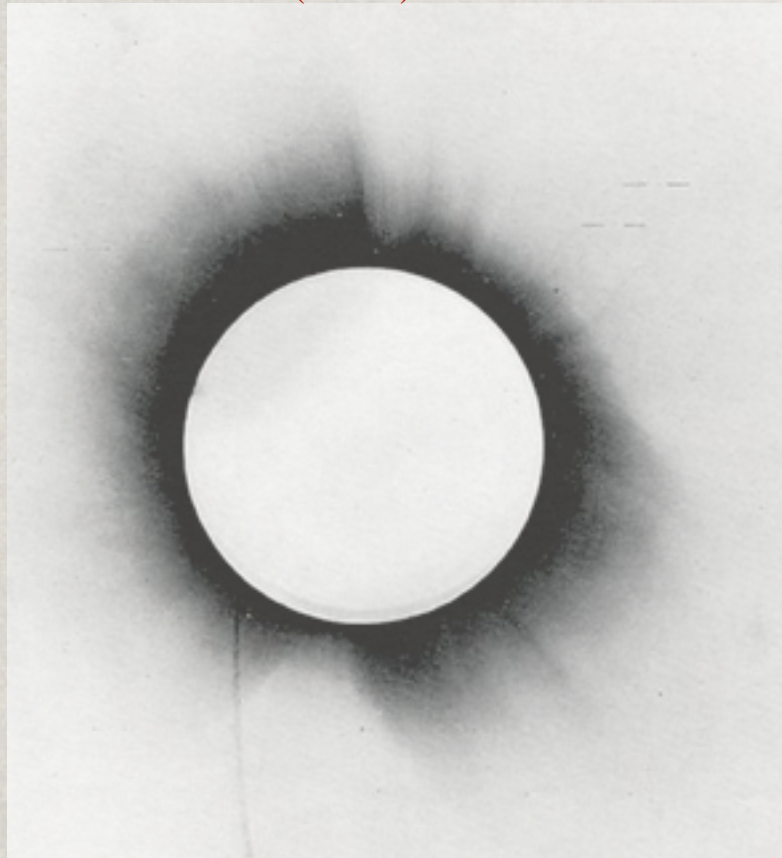


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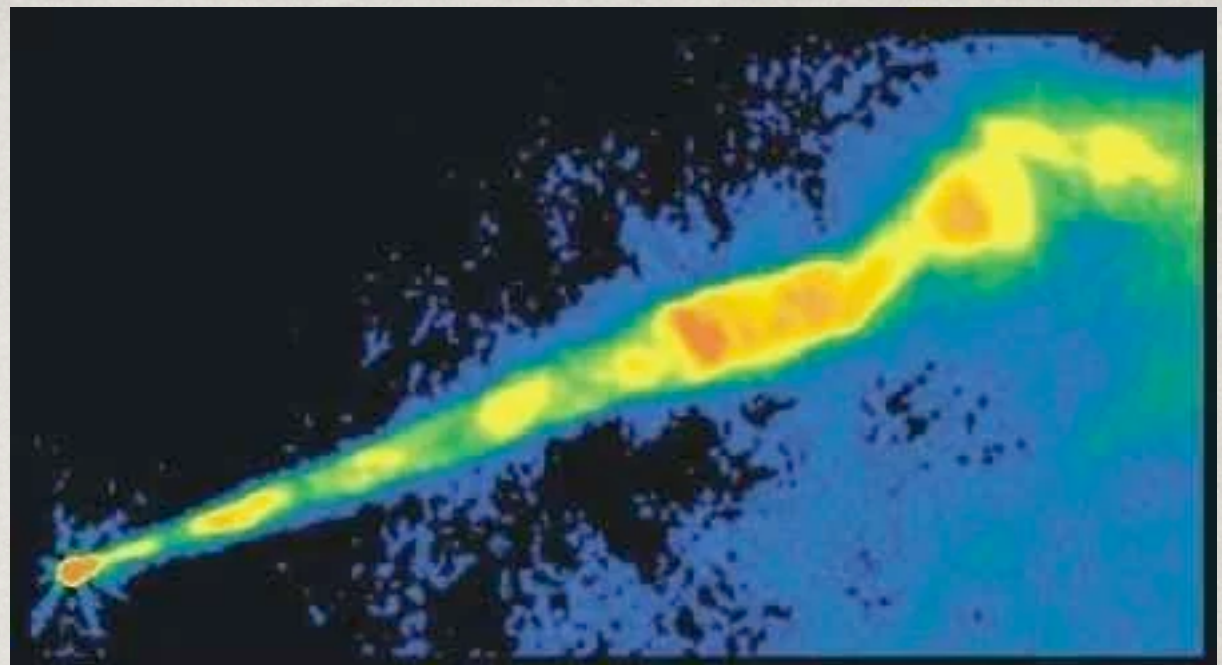


1919



2019

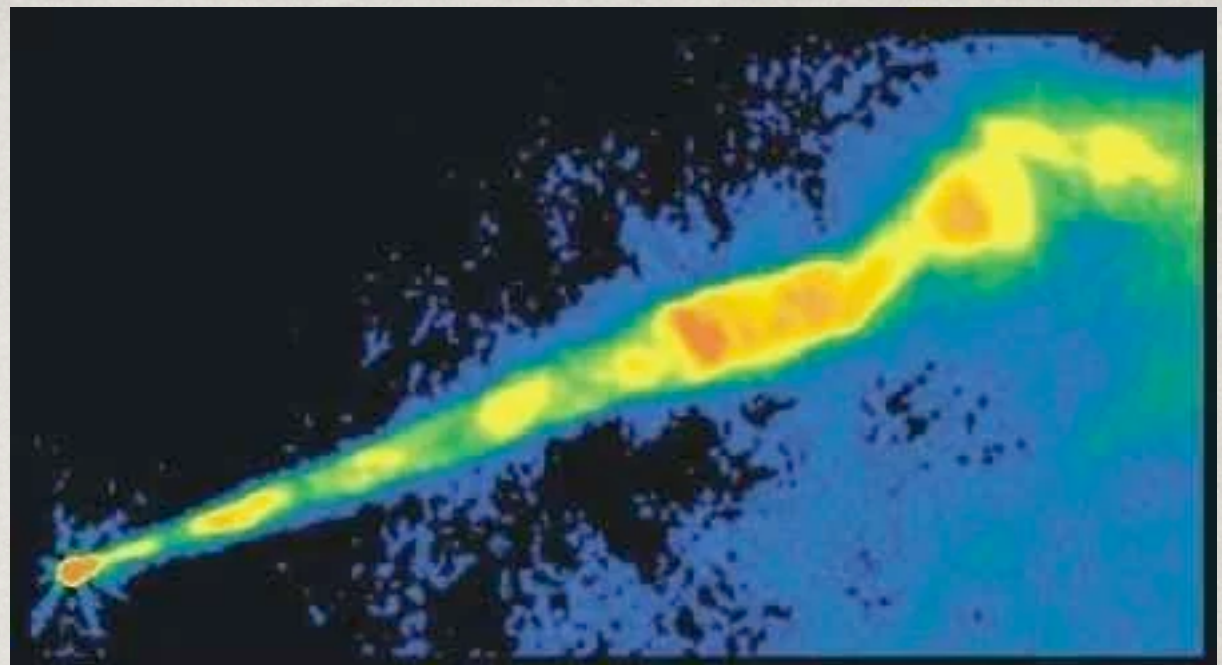
Examining strong light bending,
determined by these fundamental photon orbits,
probes the nature of the most compact objects in the universe.



M87 supermassive black hole jet
 $\sim 17^\circ$ w.r.t line of sight
(radio image - Very Large Array)

Plan: to discuss strong light bending

- 1) Paradigm: Kerr black holes
- 2) Non-Kerr (but reasonable) black holes
- 3) (Generic) horizonless ultracompact compact objects
- 4) Epilogue;

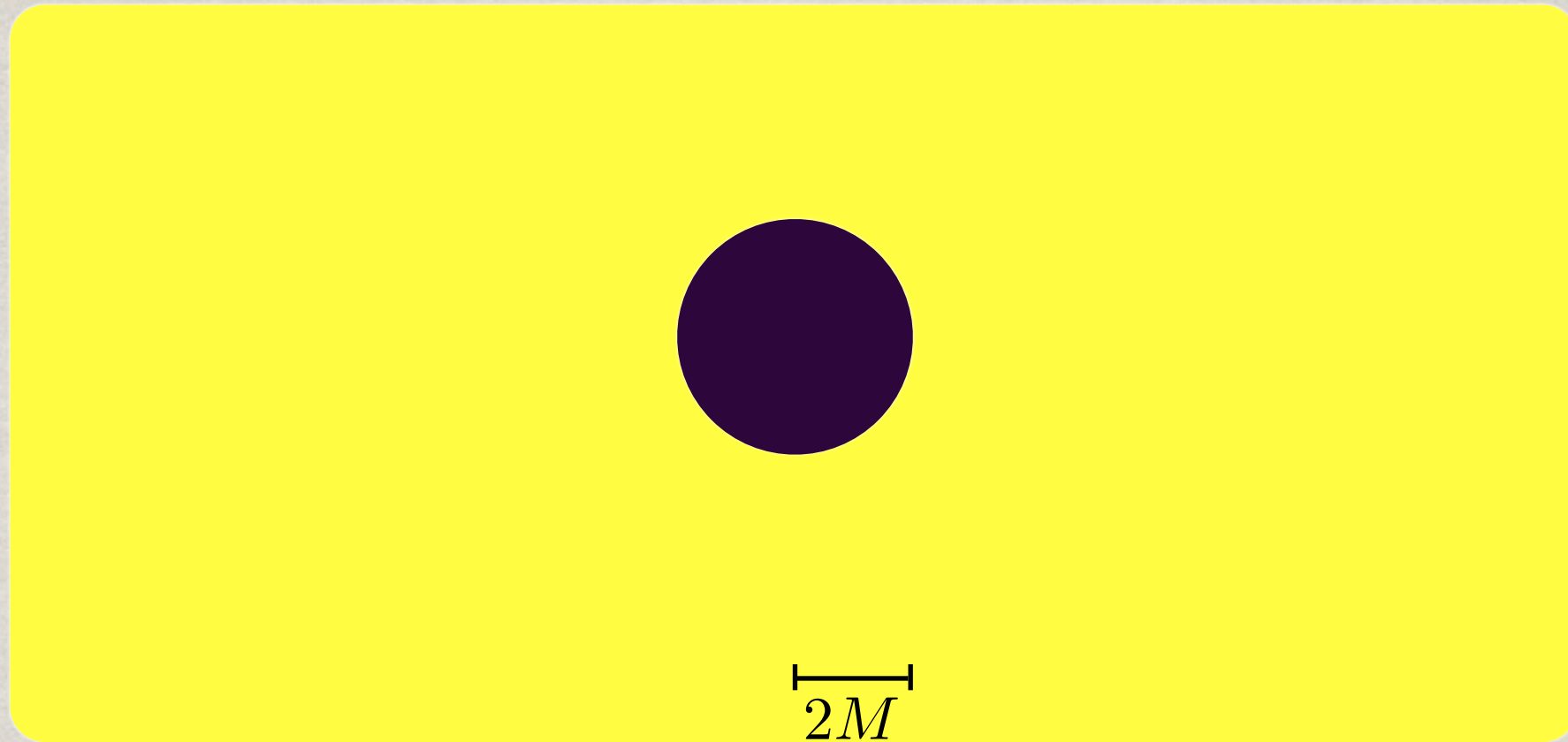


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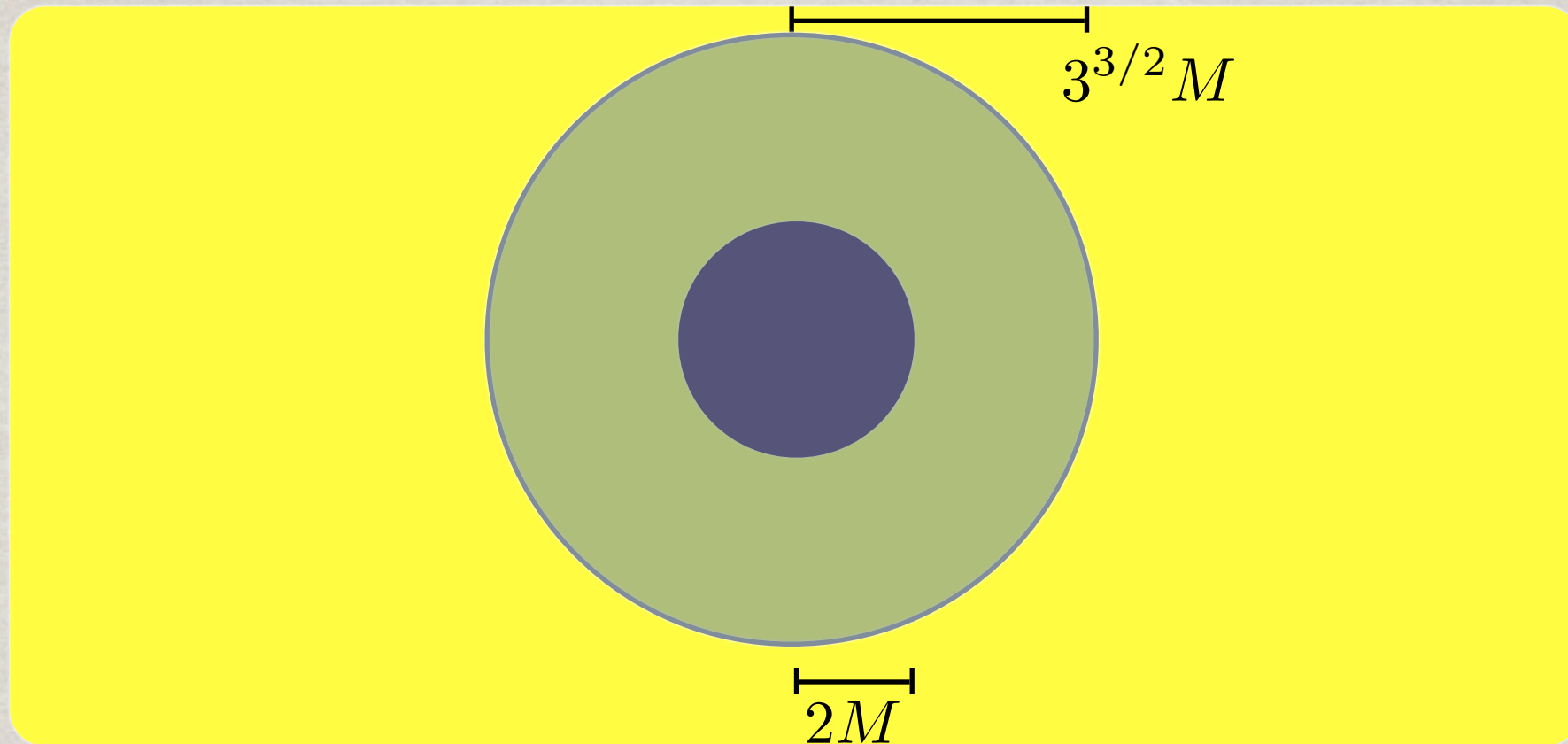
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Consider a “bright” homogeneous background with angular size much larger than the BH



Consider a “bright” homogeneous background with angular size much larger than the BH



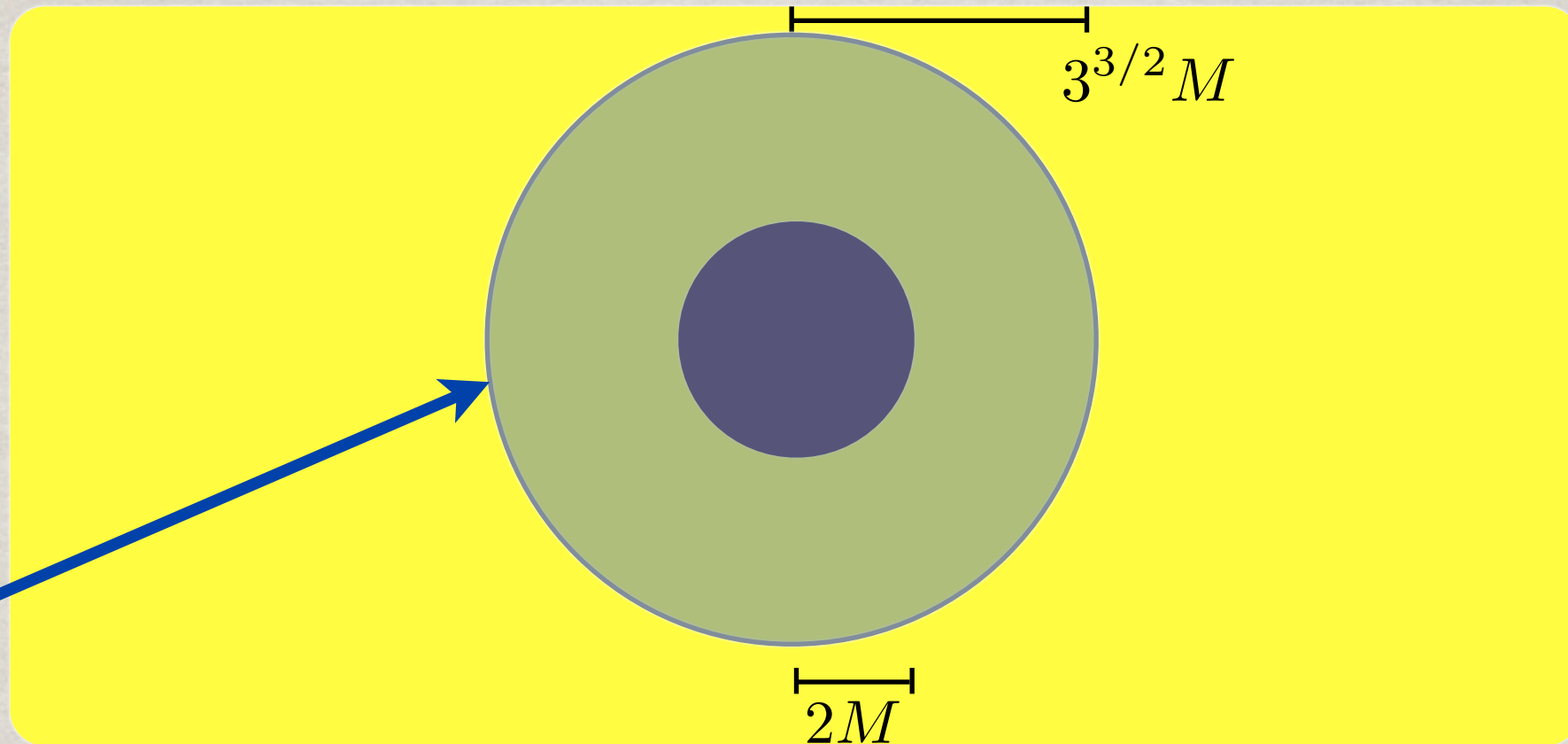
As seen by the distant observer the BH will cast a **shadow**,
larger than the horizon scale

The rim of the BH **shadow** corresponds to the lensed light ring.
The corresponding impact parameter is the critical one:

$$d \equiv \frac{j}{E} = 3^{3/2}M$$

Consider a “bright” homogeneous background with angular size much larger than the BH

Determined
by (planar)
light rings



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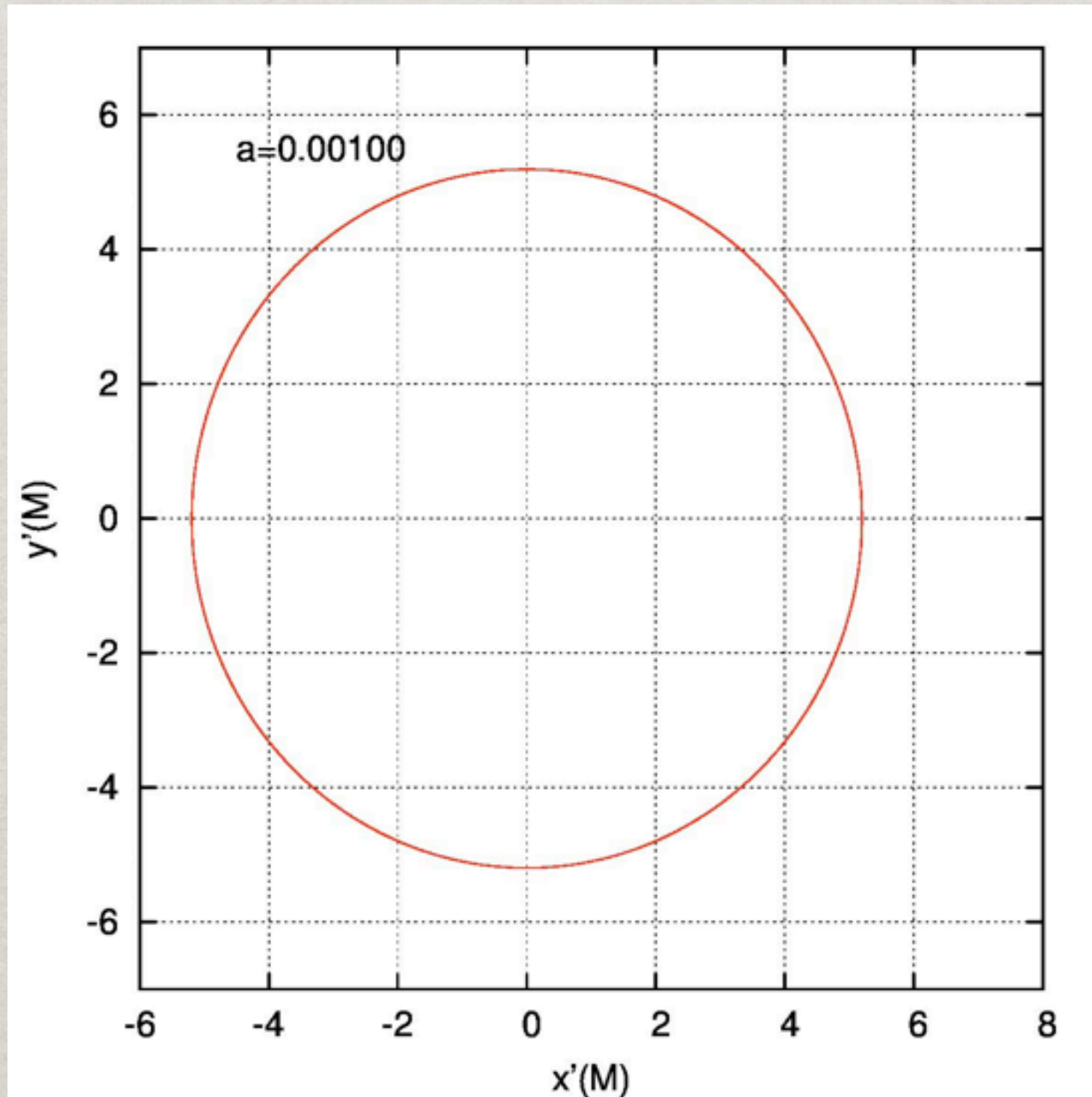
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Shadow of a Kerr black hole:

(equatorial plane observation)

J. Bardeen (1973)

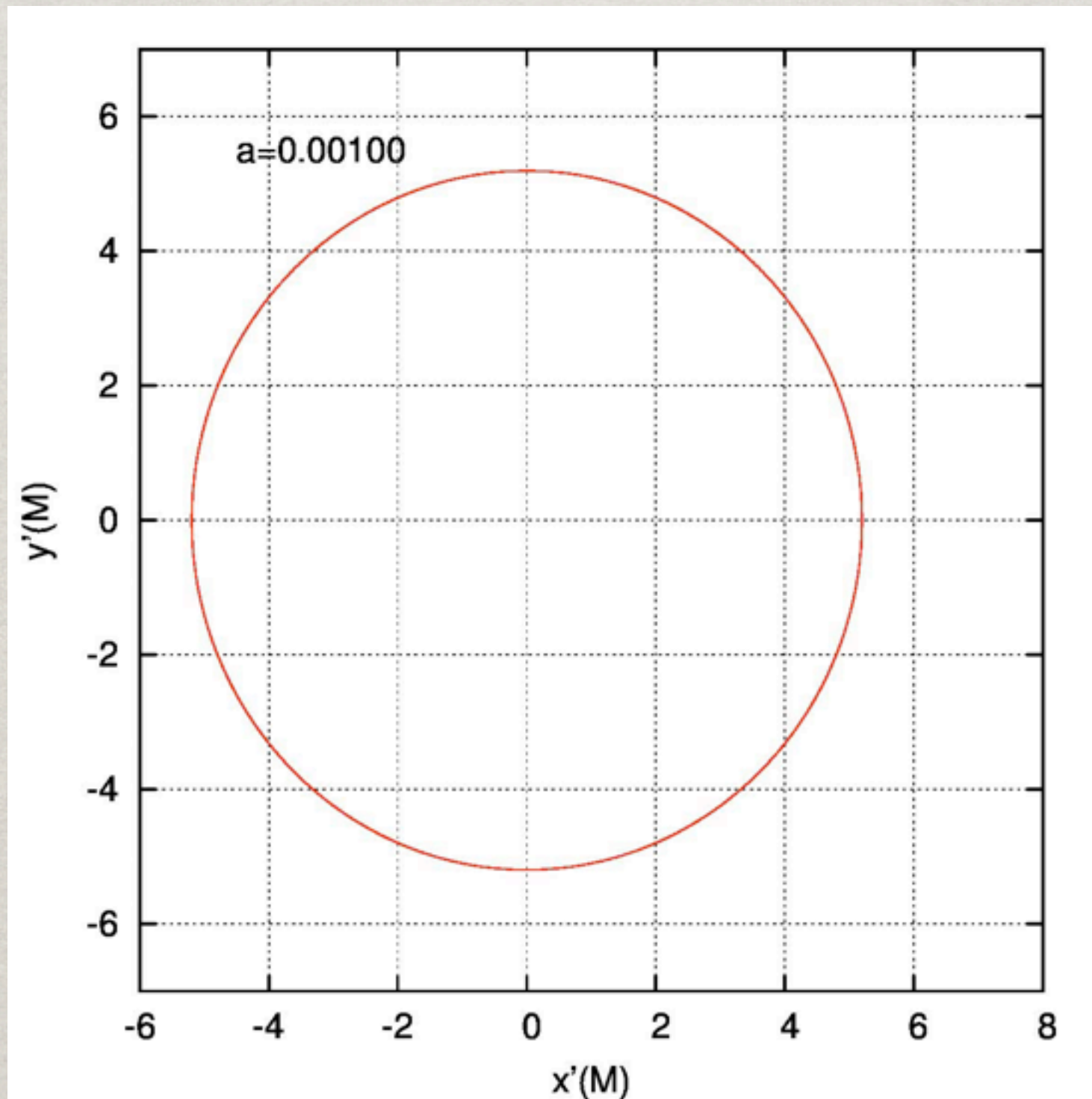


↑
Spin
axis

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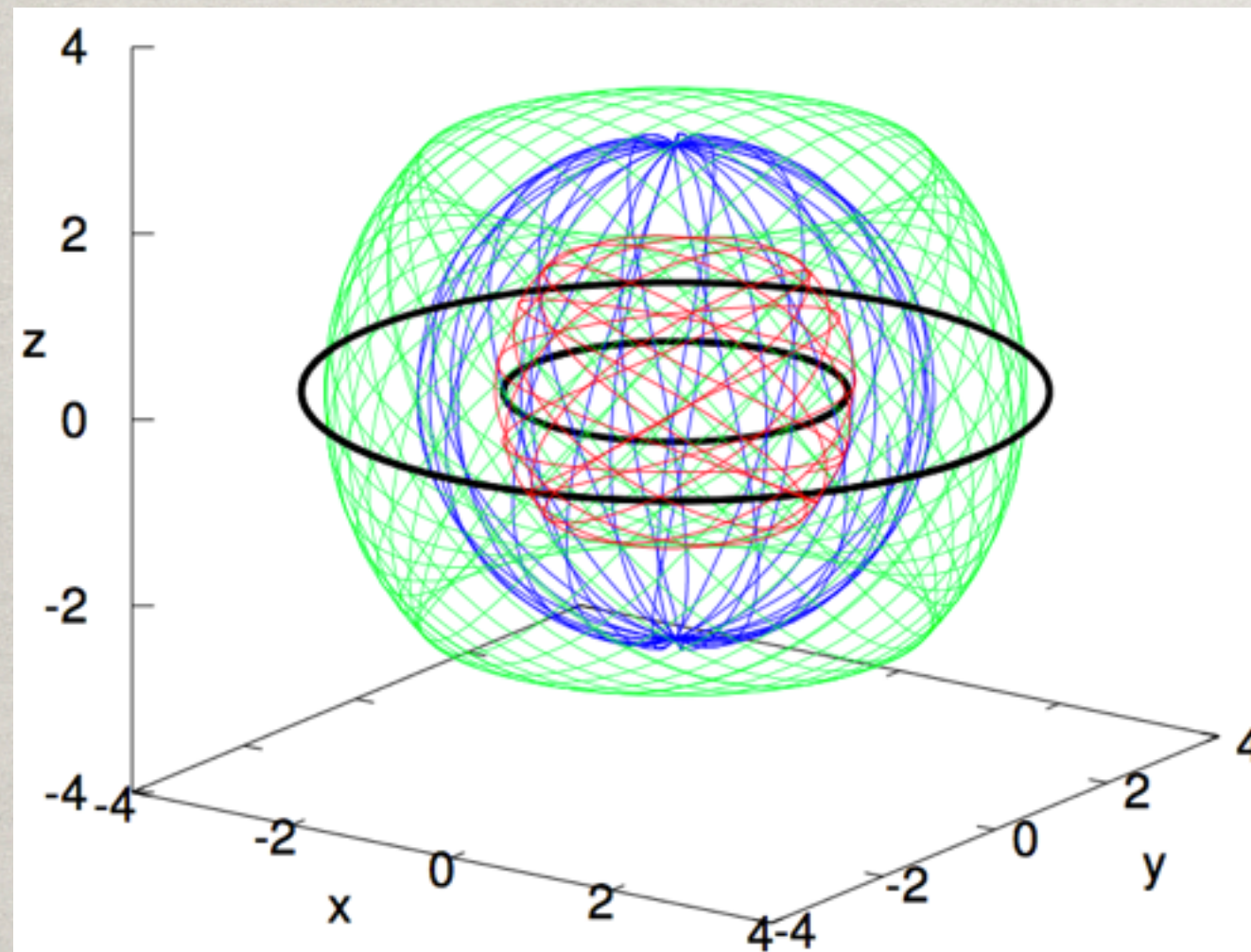
The edge of the shadow is determined by the

Fundamental Photon Orbits

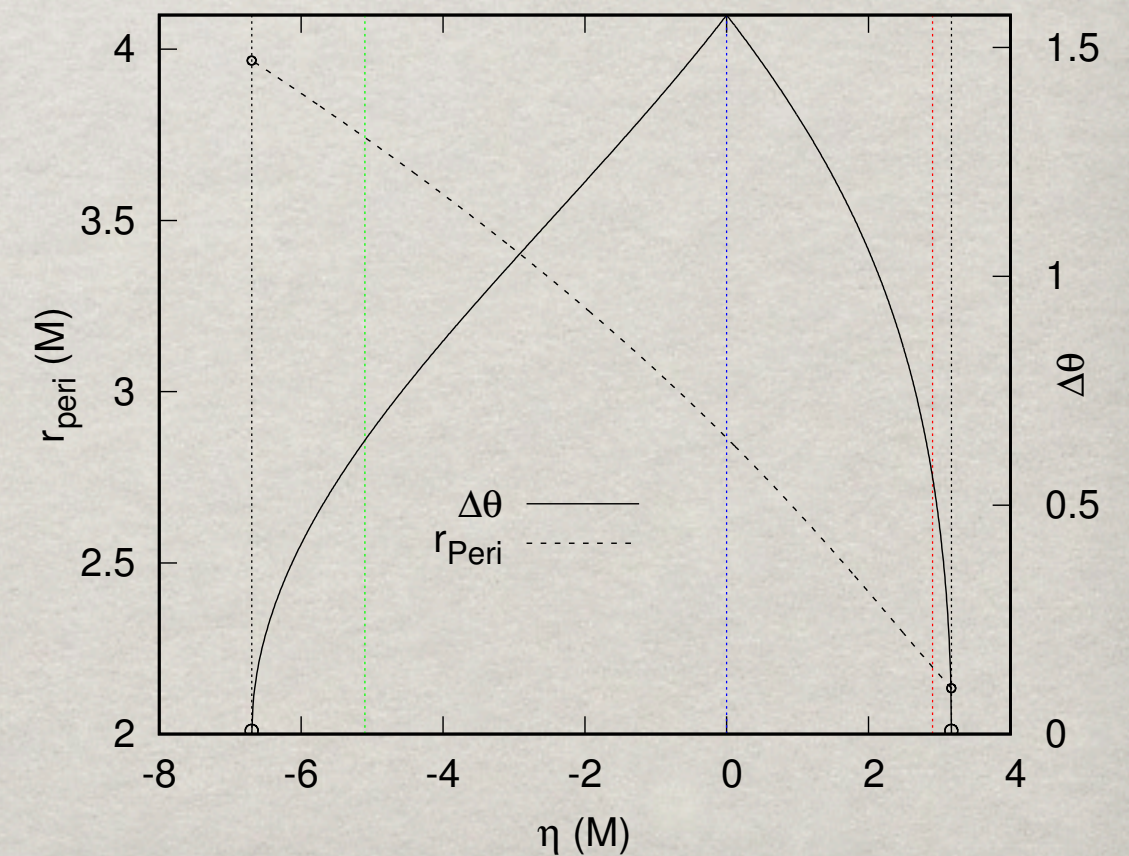
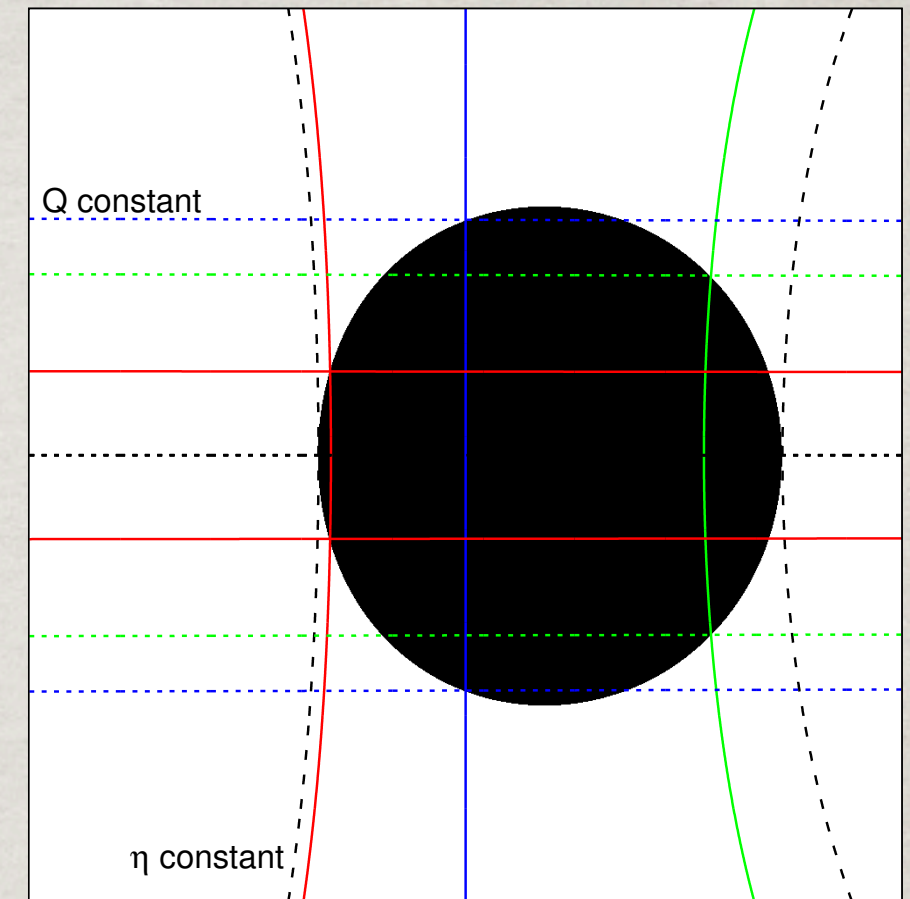
Cunha, C.H., Radu, PRD 96 (2017) 024039

Called spherical orbits in Kerr case

Teo, GRG 35 (2003) 1909



$j \sim 0.82$



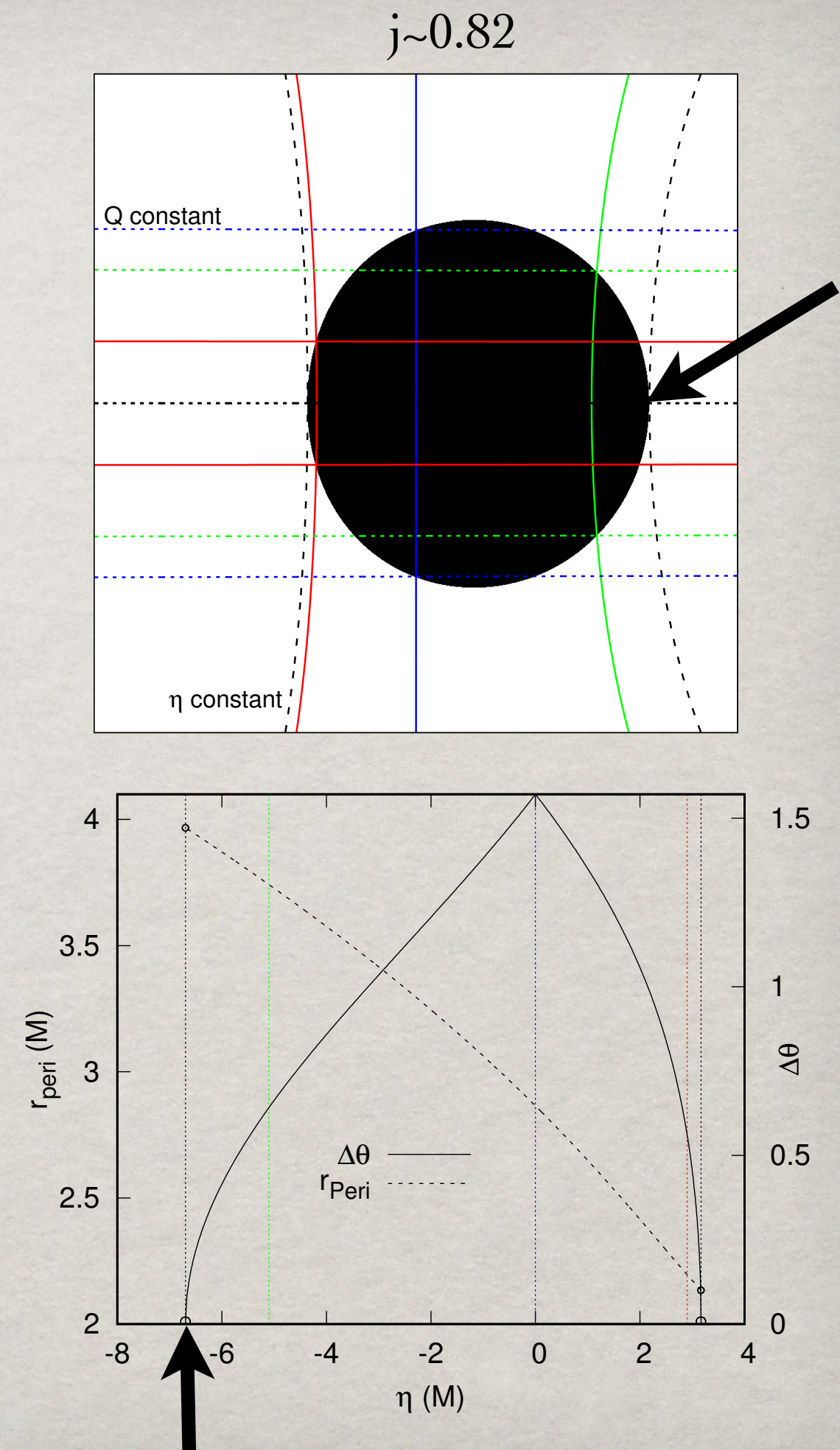
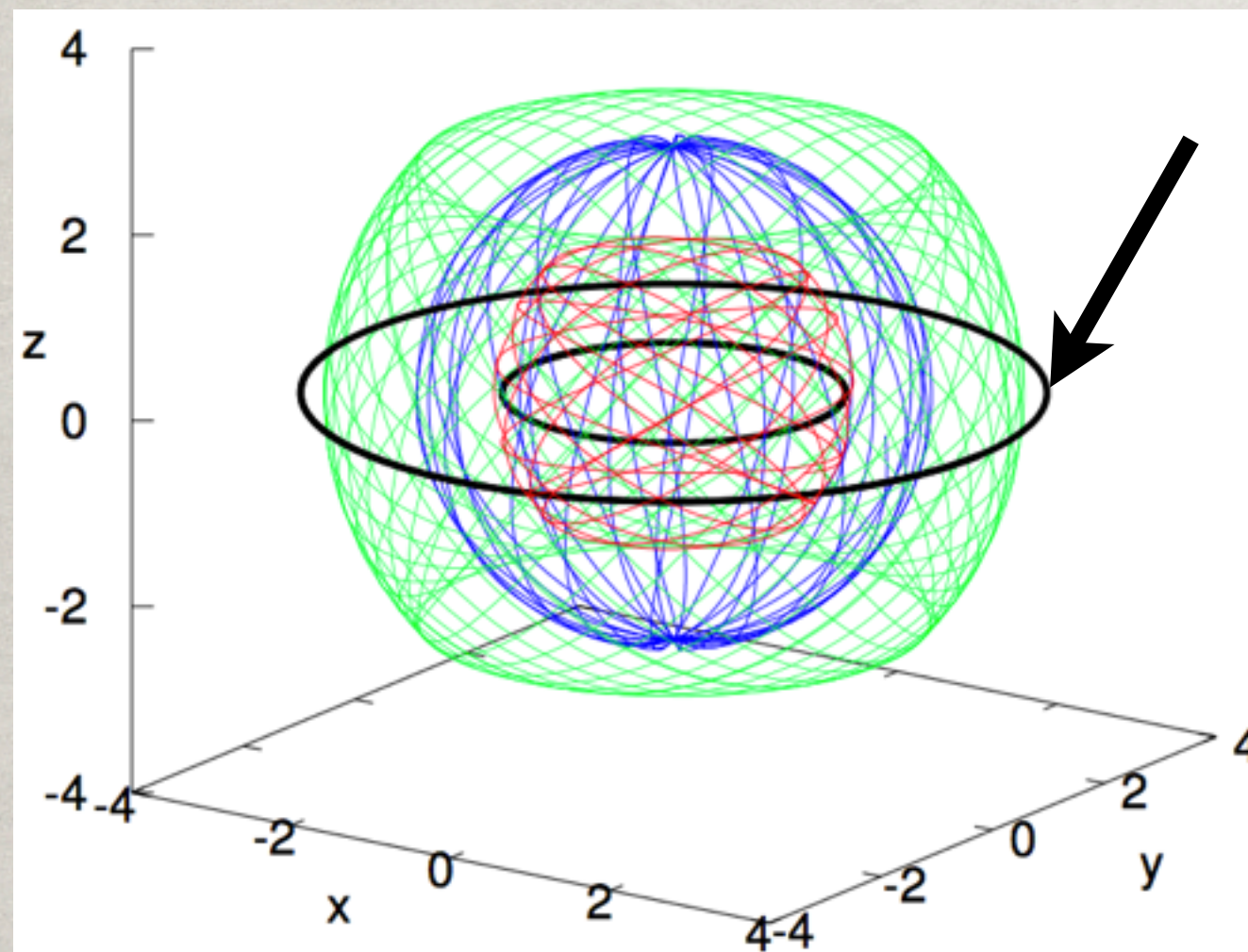
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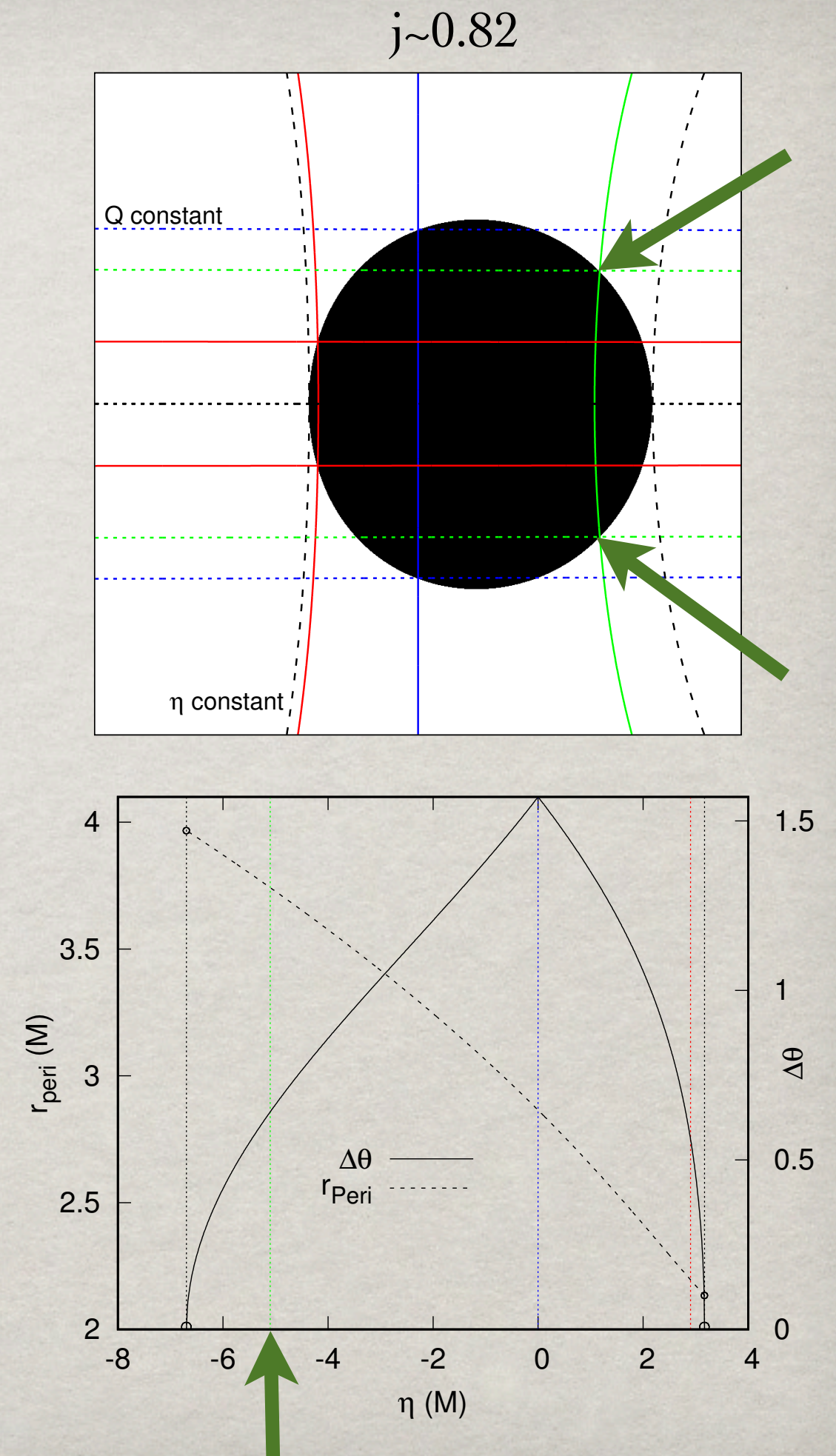
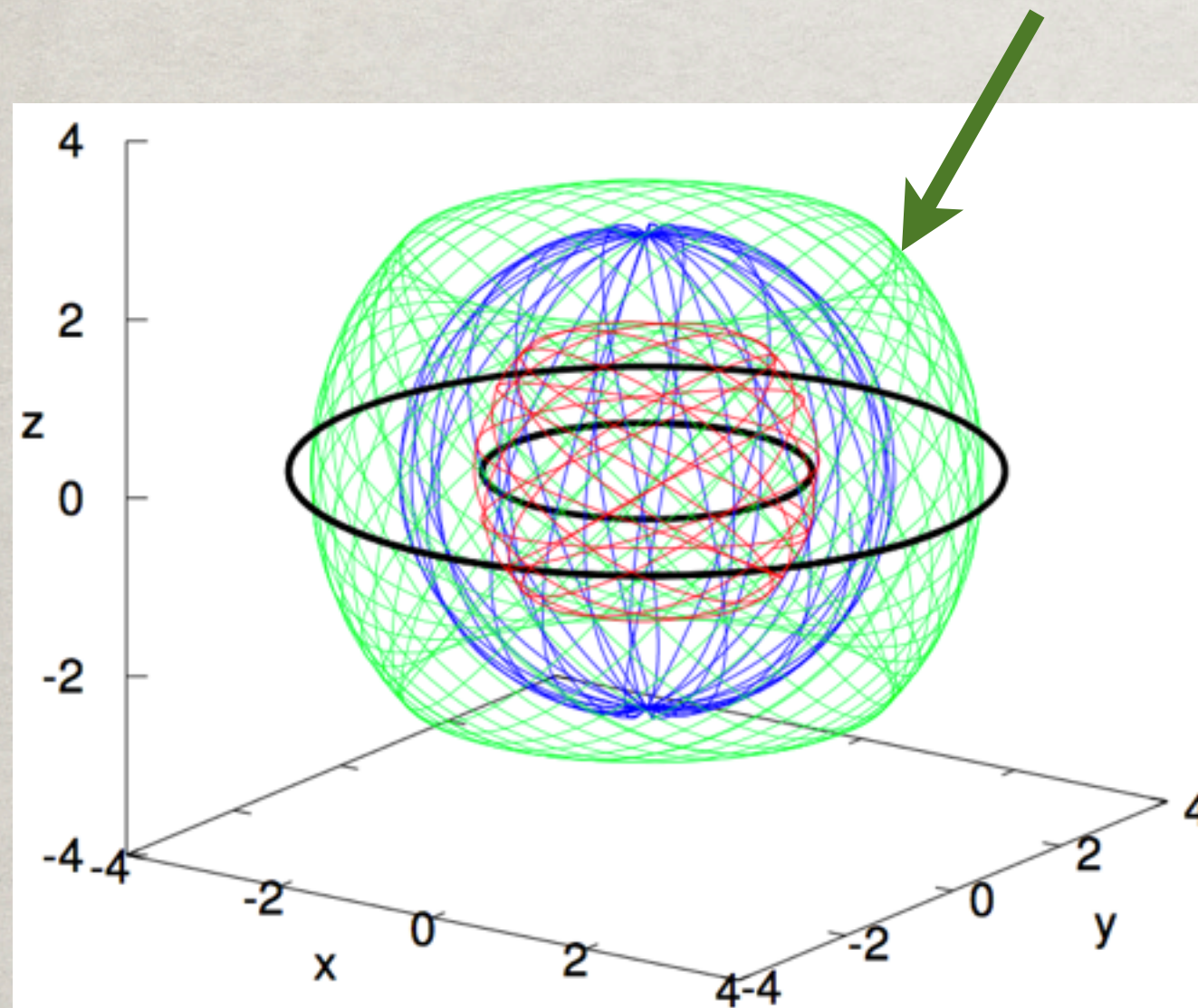
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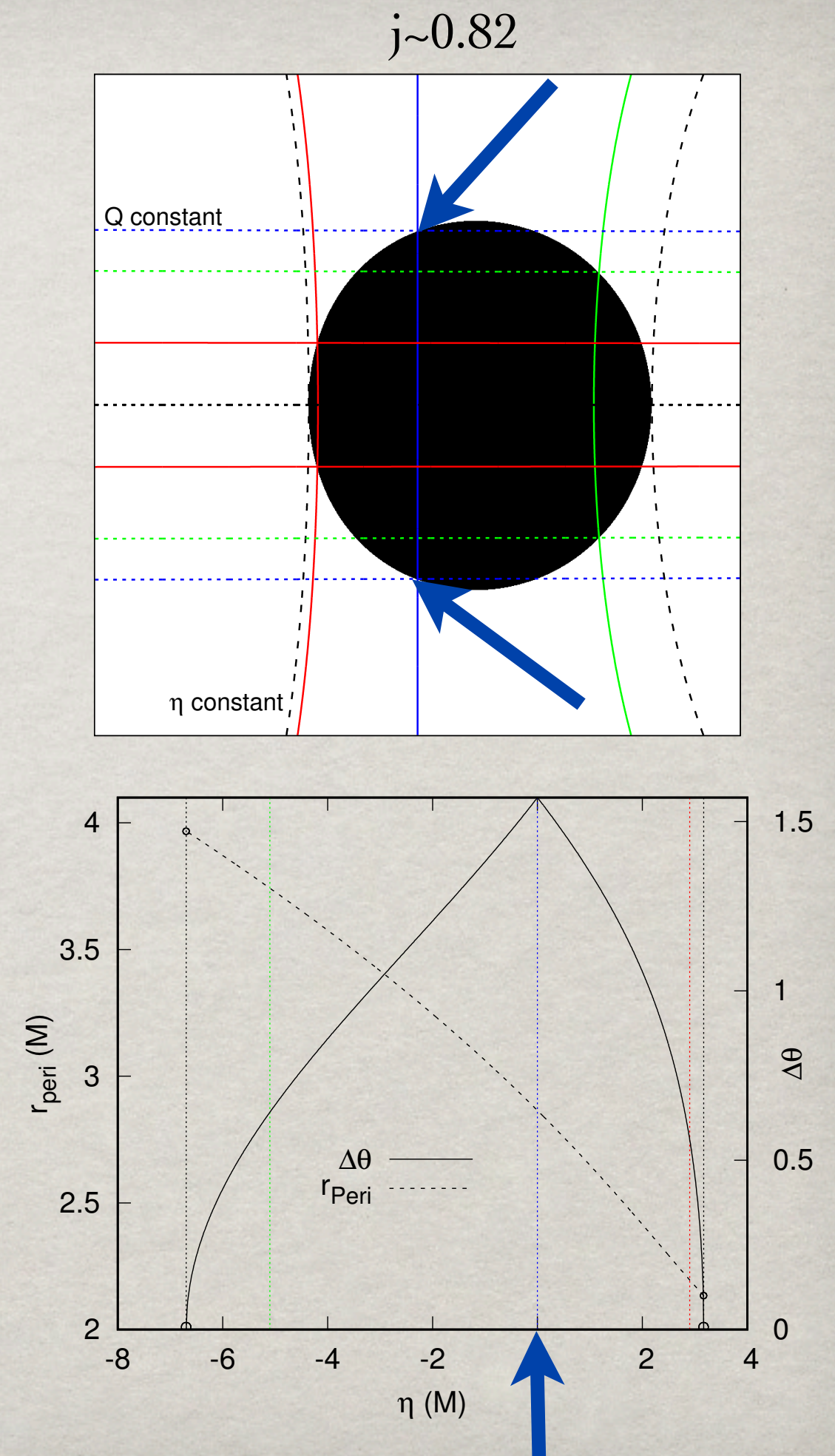
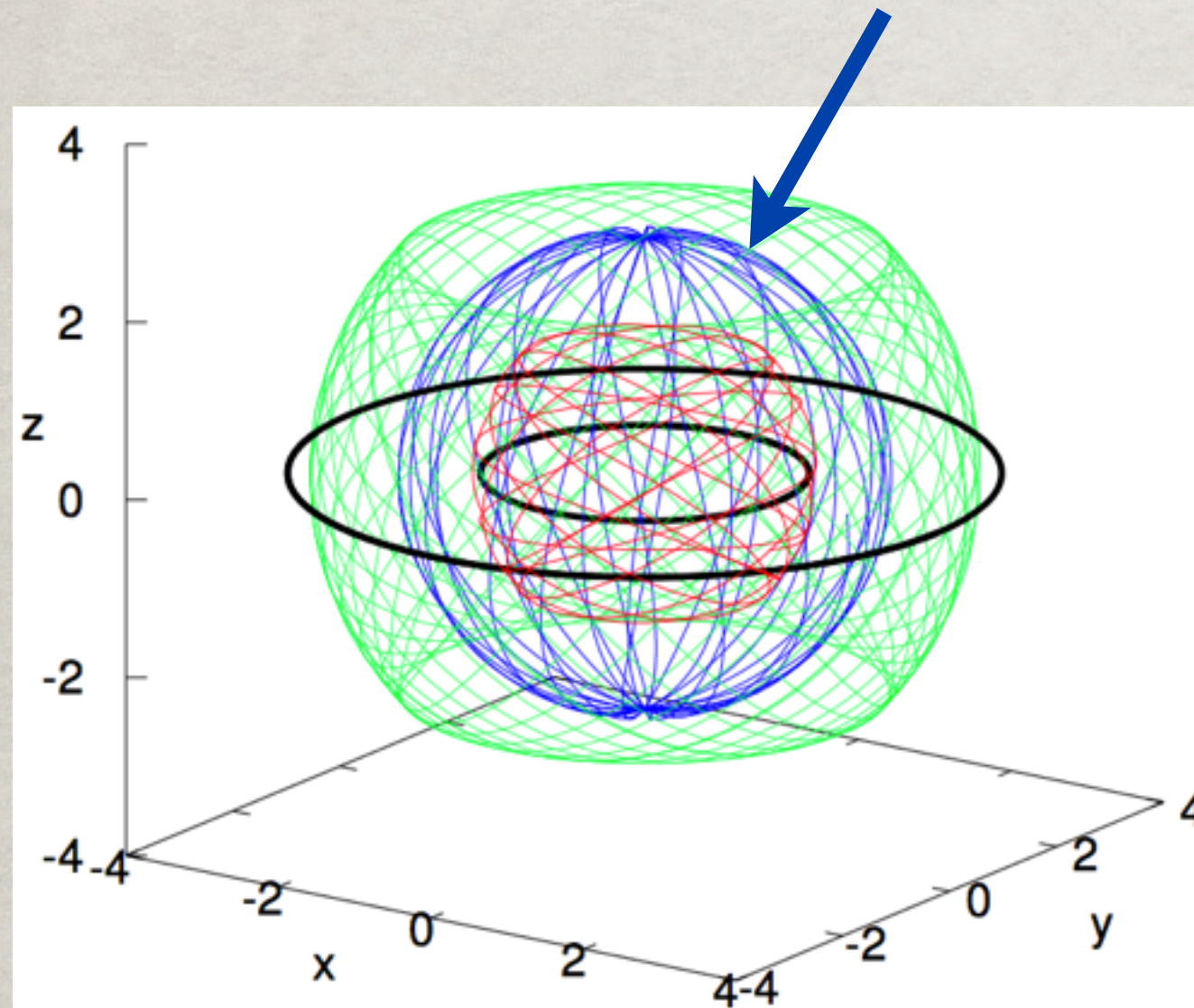
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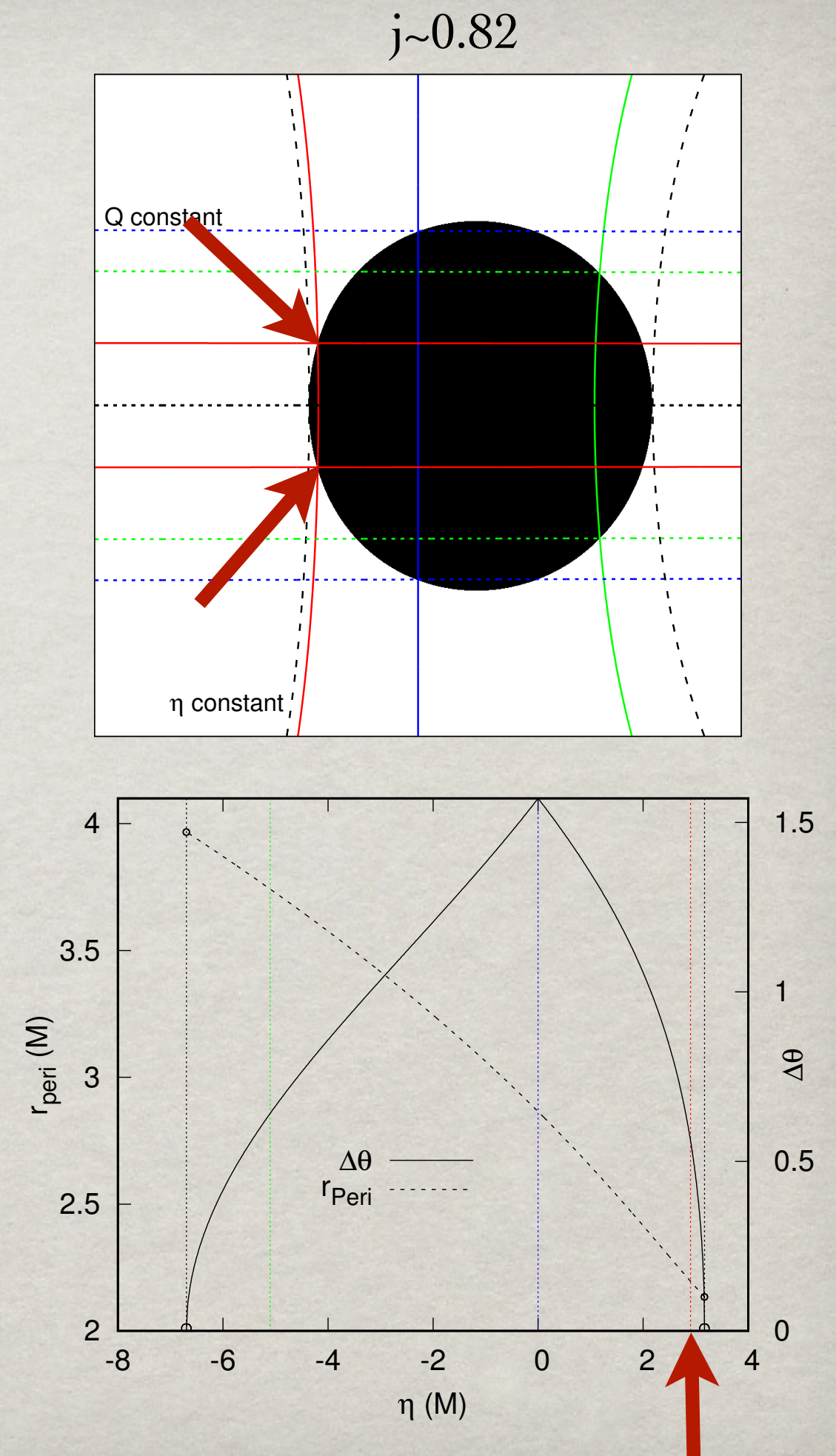
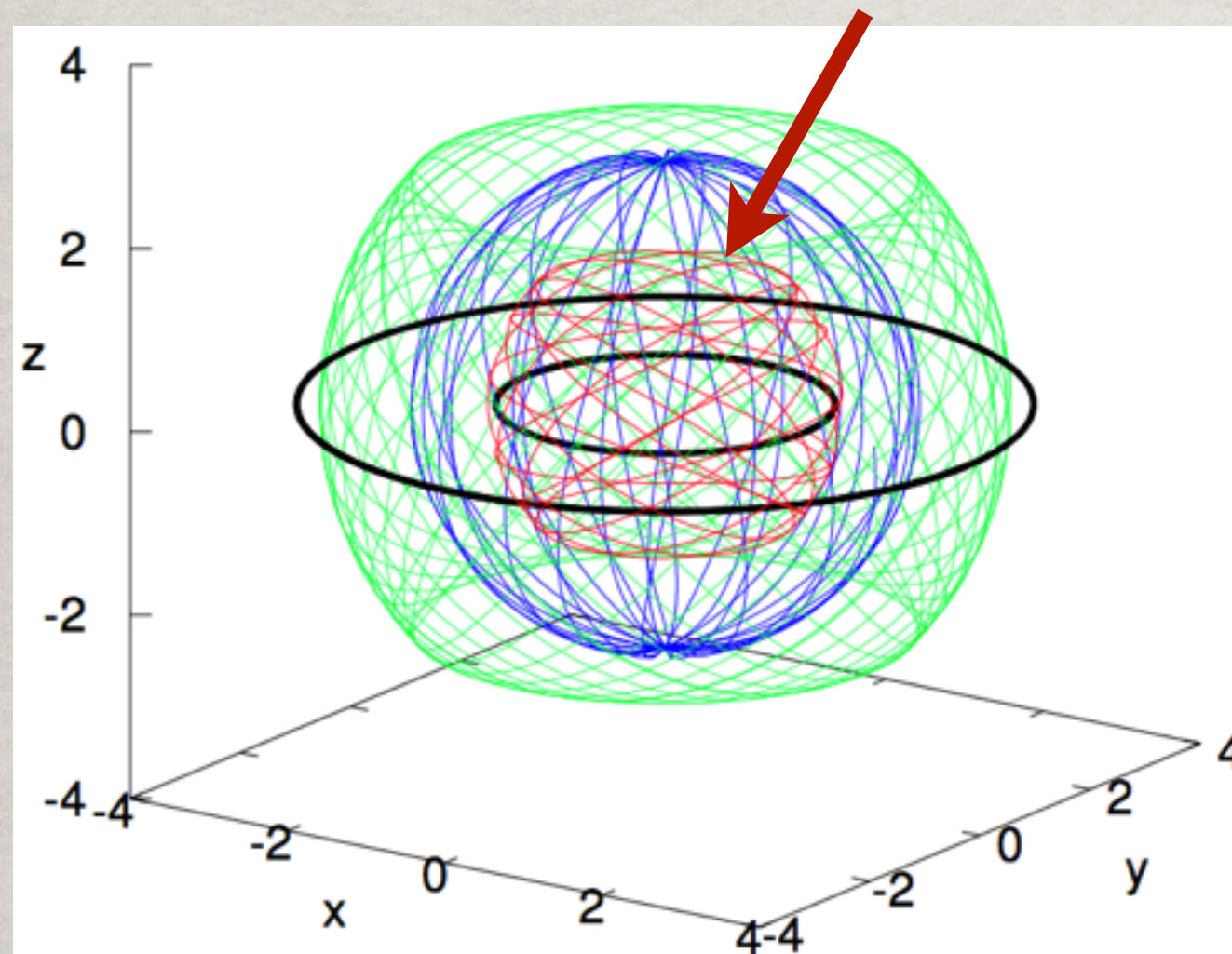
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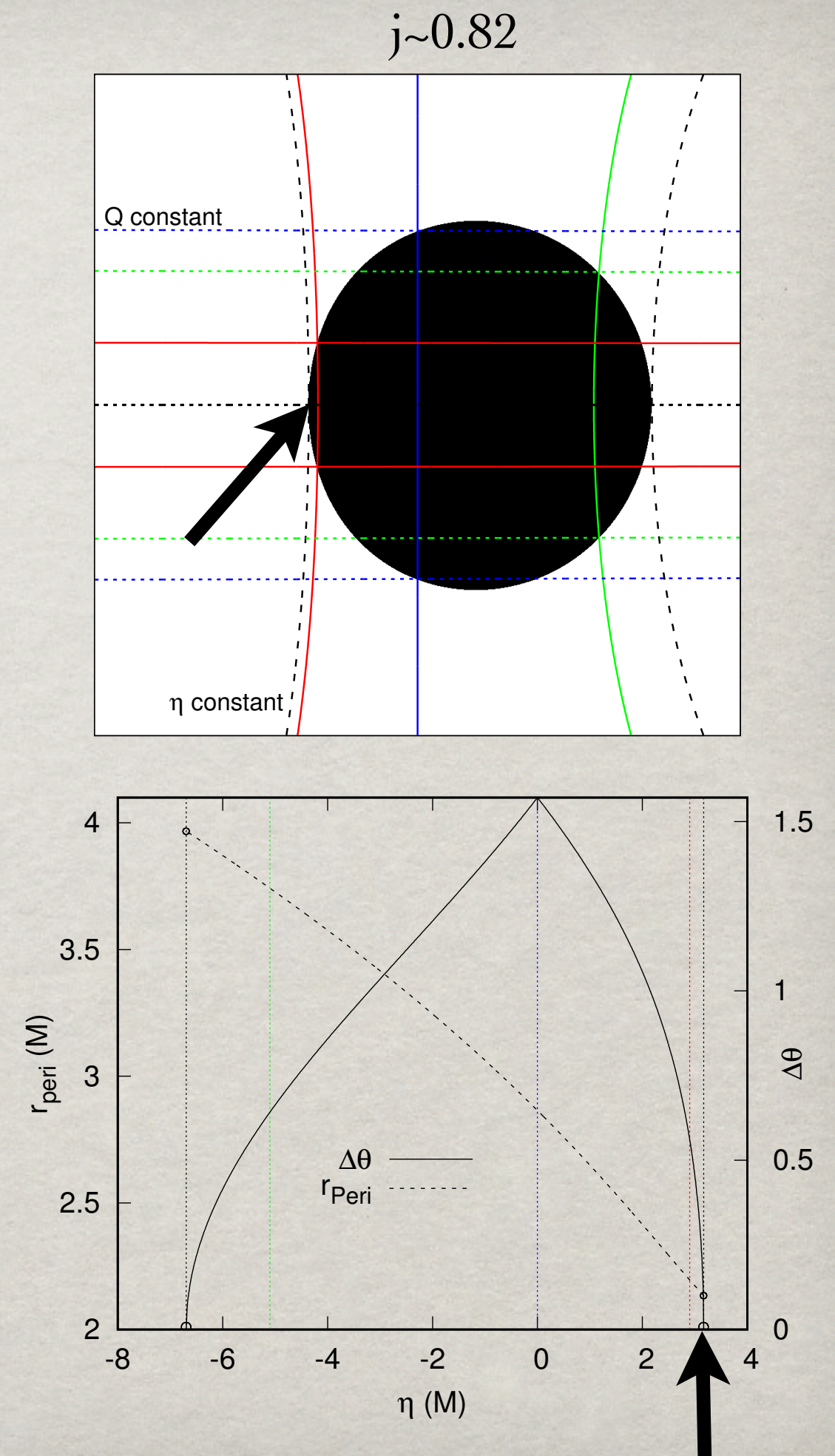
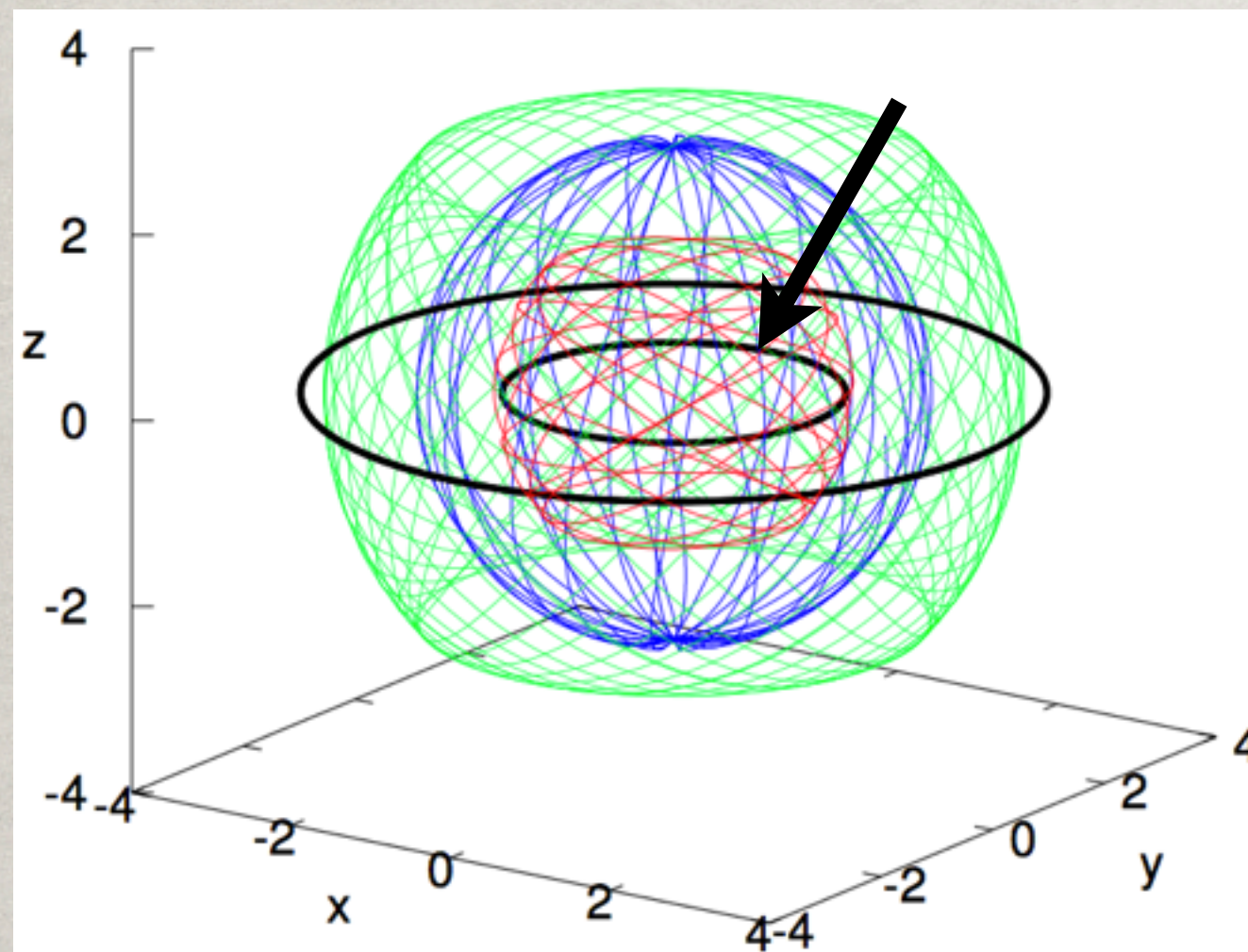
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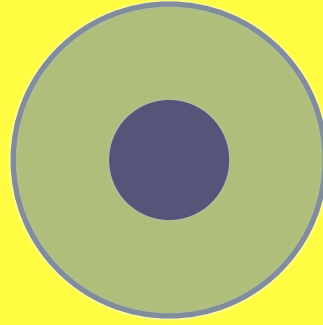
In a more astrophysical scenario:

background light is replaced
by synchrotron radiation from
accretion disk

Cunningham and Bardeen, (1970s)

J. P. Luminet (1979)

Academic



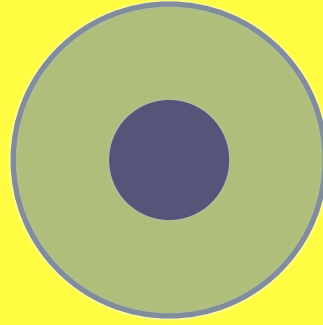
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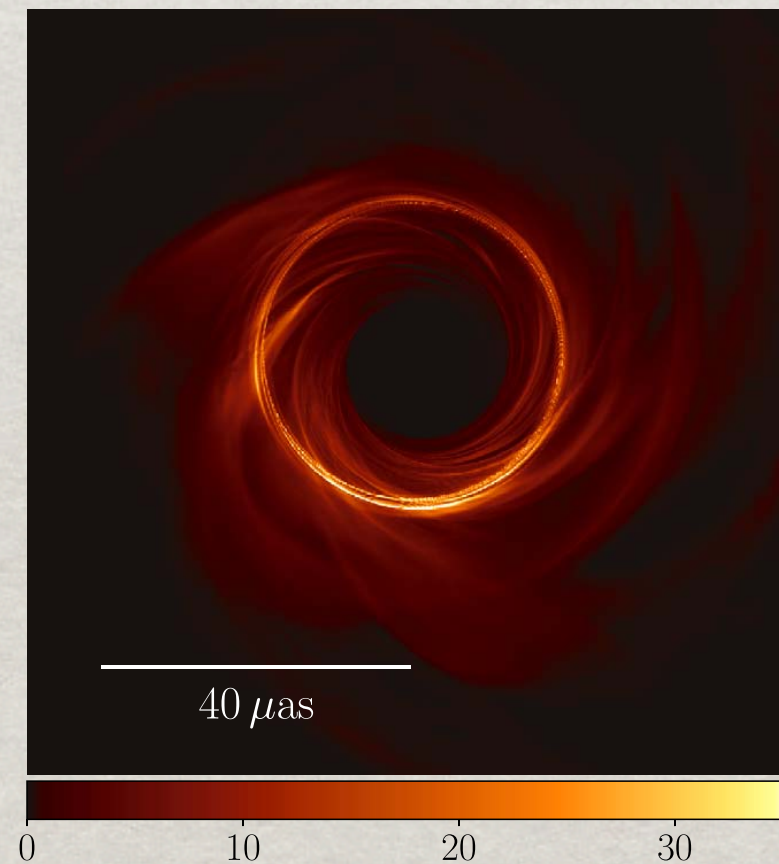
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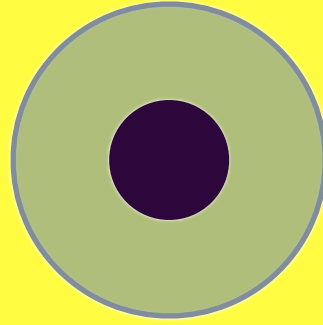


Synthetic images are generated
by General Relativistic
Magneto-Hydrodynamics
(GRMHD)
simulations
using the Kerr metric

GRMHD



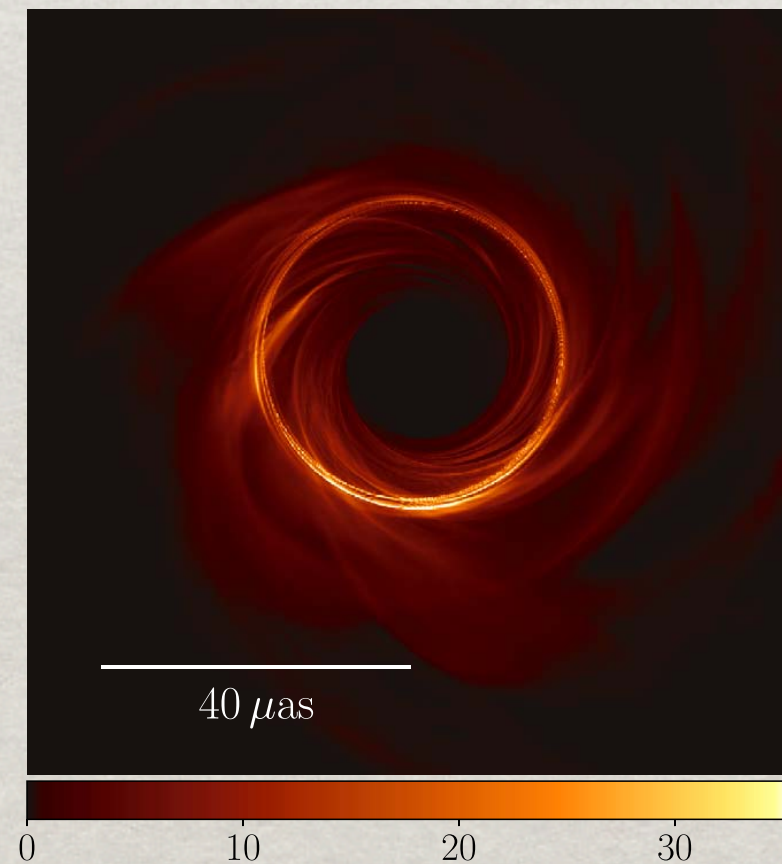
Academic



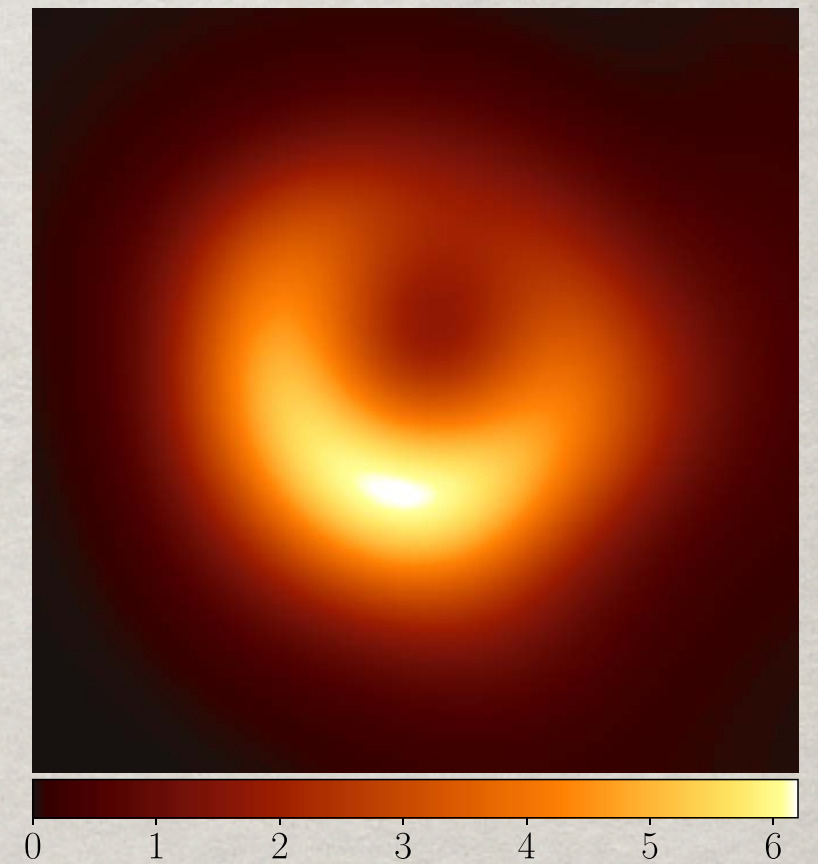
A Gaussian Blurring filter
is applied to a synthetic image
to reproduce
real EHT observations

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GRMHD

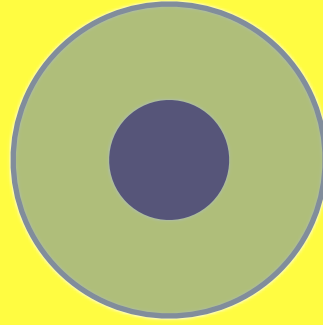


Blurred GRMHD



The synthetic blurred image
is similar do real data,
consistent with a Kerr black hole

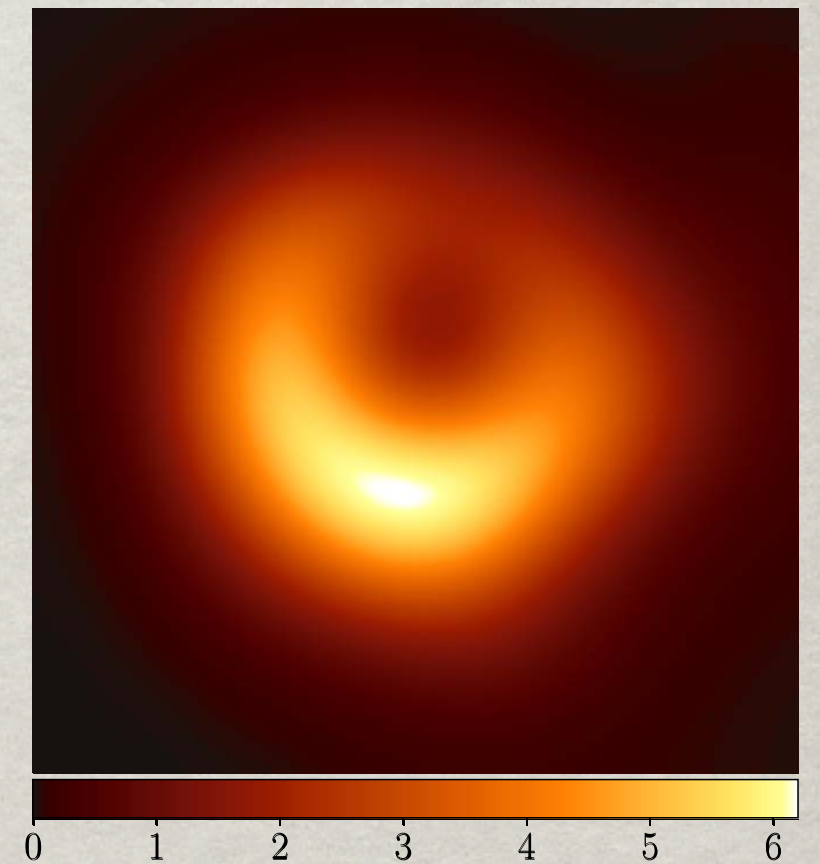
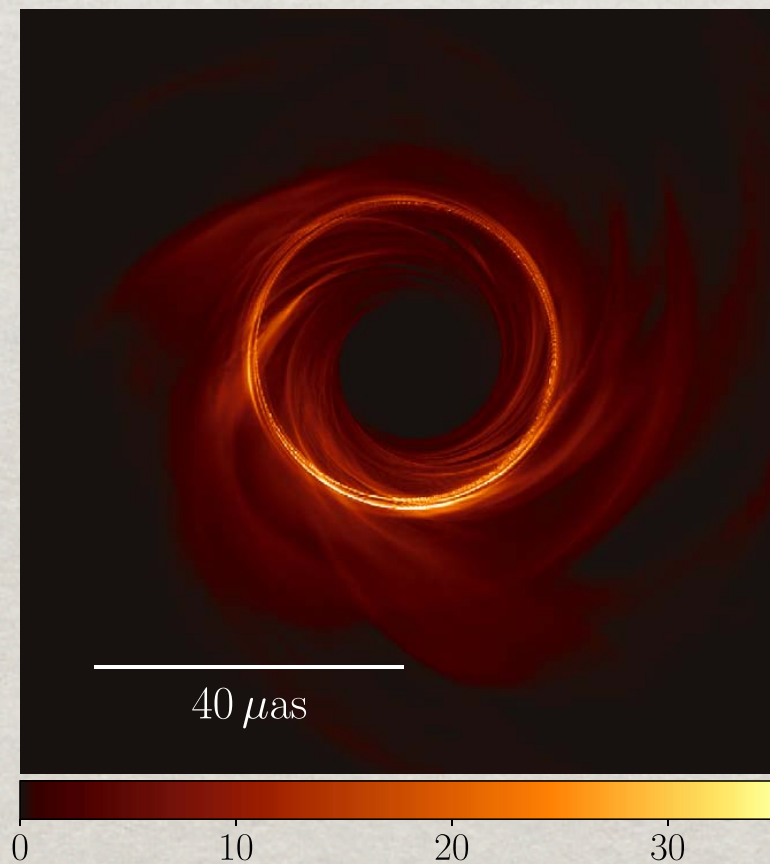
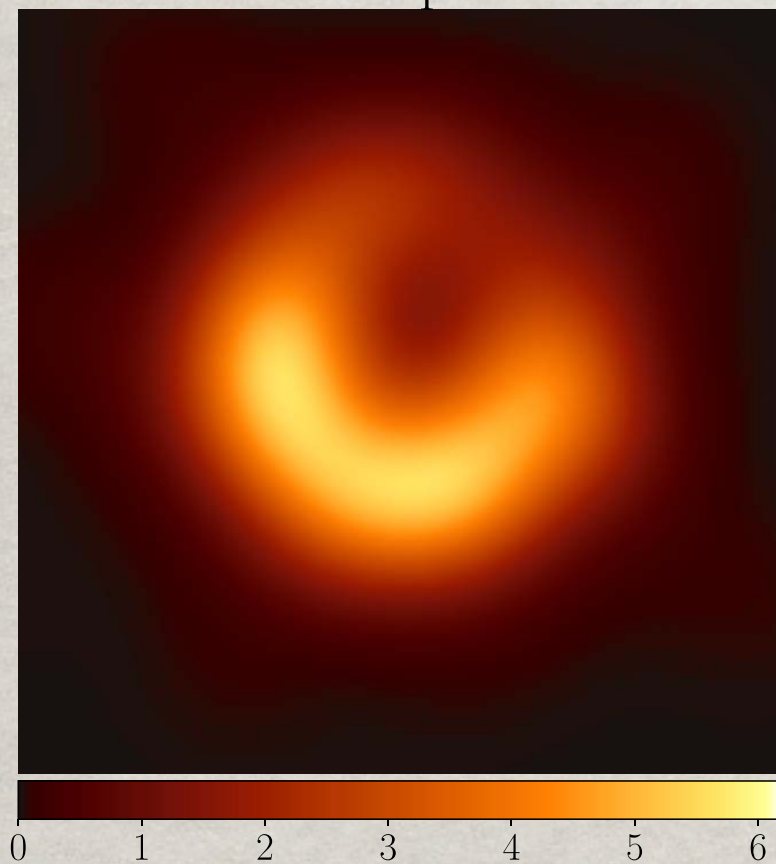
Academic



M87 April 6

GRMHD

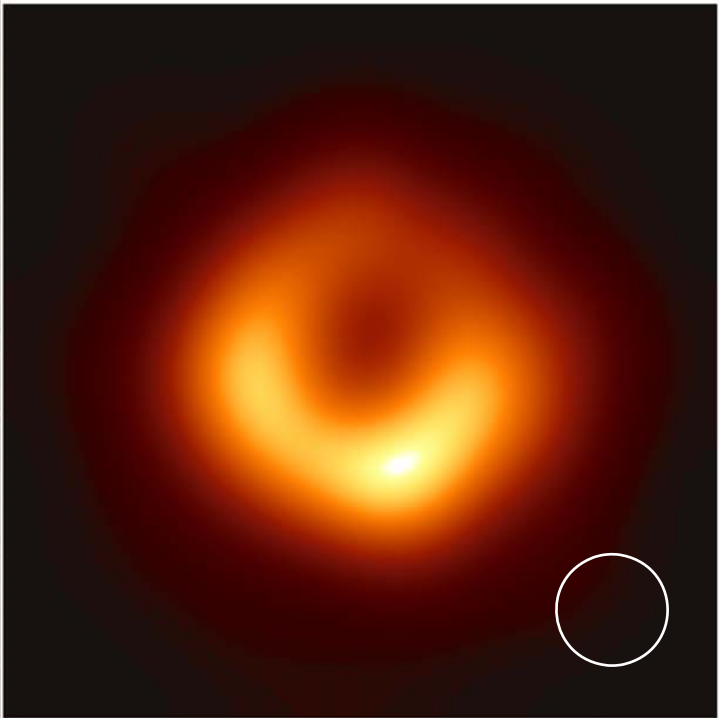
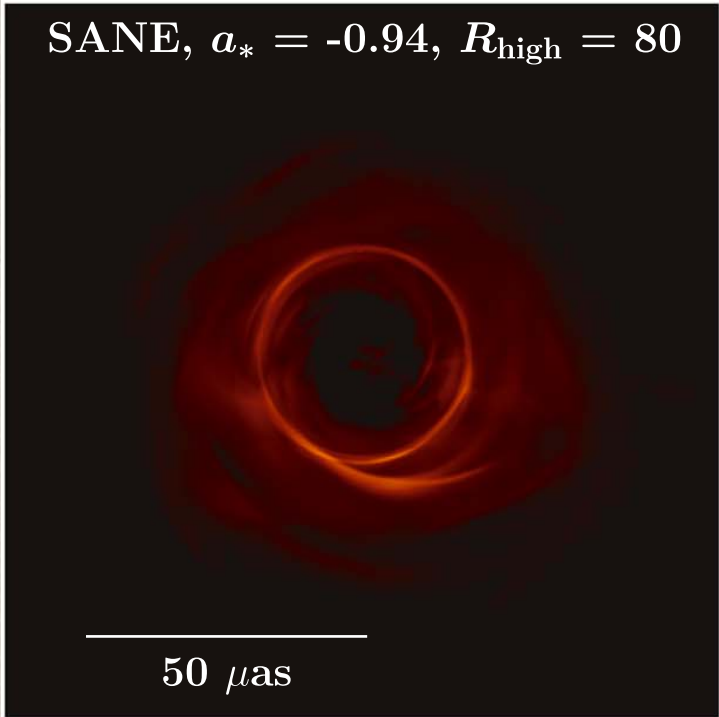
Blurred GRMHD



Brightness Temperature (10^9 K)

Figure 1. Left panel: an EHT2017 image of M87 from Paper [IV](#) of this series (see their Figure 15). Middle panel: a simulated image based on a GRMHD model. Right panel: the model image convolved with a $20 \mu\text{as}$ FWHM Gaussian beam. Although the most evident features of the model and data are similar, fine features in the model are not resolved by EHT.

GRMHD models



Schwarzschild based
estimate of the light ring
sky angle:

Table 1
Parameters of M87*

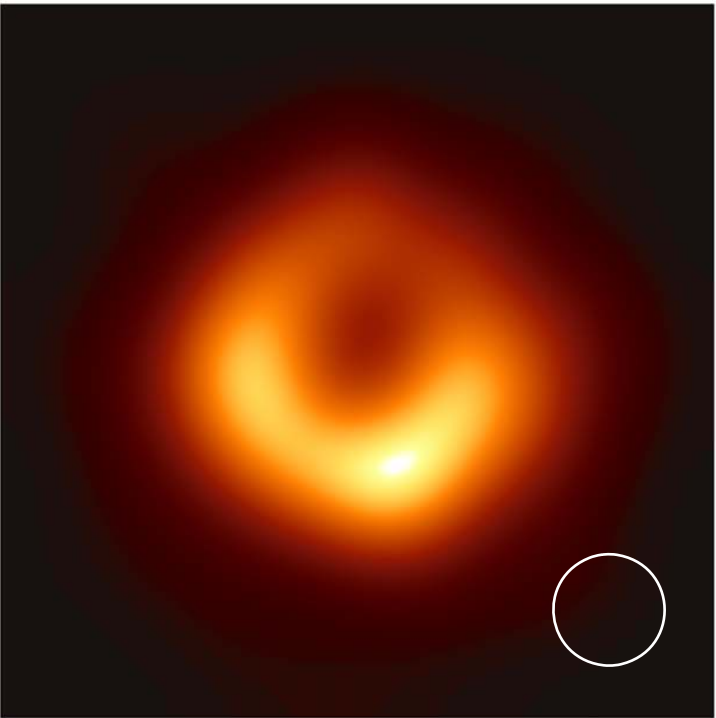
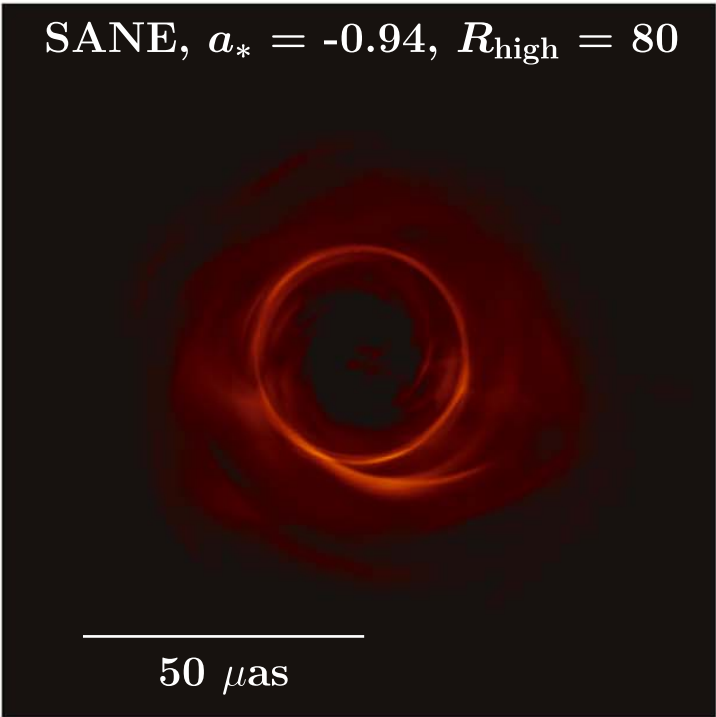
Parameter	Estimate
Ring diameter ^a d	$42 \pm 3 \mu\text{as}$
Ring width ^a	$< 20 \mu\text{as}$
Crescent contrast ^b	$> 10:1$
Axial ratio ^a	$< 4:3$
Orientation PA	$150^\circ\text{--}200^\circ$ east of north
$\theta_g = GM/Dc^2$ ^c	$3.8 \pm 0.4 \mu\text{as}$
$\alpha = d/\theta_g$ ^d	$11^{+0.5}_{-0.3}$
M ^c	$(6.5 \pm 0.7) \times 10^9 M_\odot$
Parameter	Prior Estimate
D ^e	$(16.8 \pm 0.8) \text{ Mpc}$
$M(\text{stars})$ ^e	$6.2^{+1.1}_{-0.6} \times 10^9 M_\odot$
$M(\text{gas})$ ^e	$3.5^{+0.9}_{-0.3} \times 10^9 M_\odot$

Notes.

- ^a Derived from the image domain.
^b Derived from crescent model fitting.
^c The mass and systematic errors are averages of the three methods (geometric models, GRMHD models, and image domain ring extraction).
^d The exact value depends on the method used to extract d , which is reflected in the range given.
^e Rederived from likelihood distributions (Paper VI).

$$2 \times \sqrt{27} \times 3.8 \mu\text{as} \simeq 39.5 \mu\text{as}$$

GRMHD models



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estimate of the light ring
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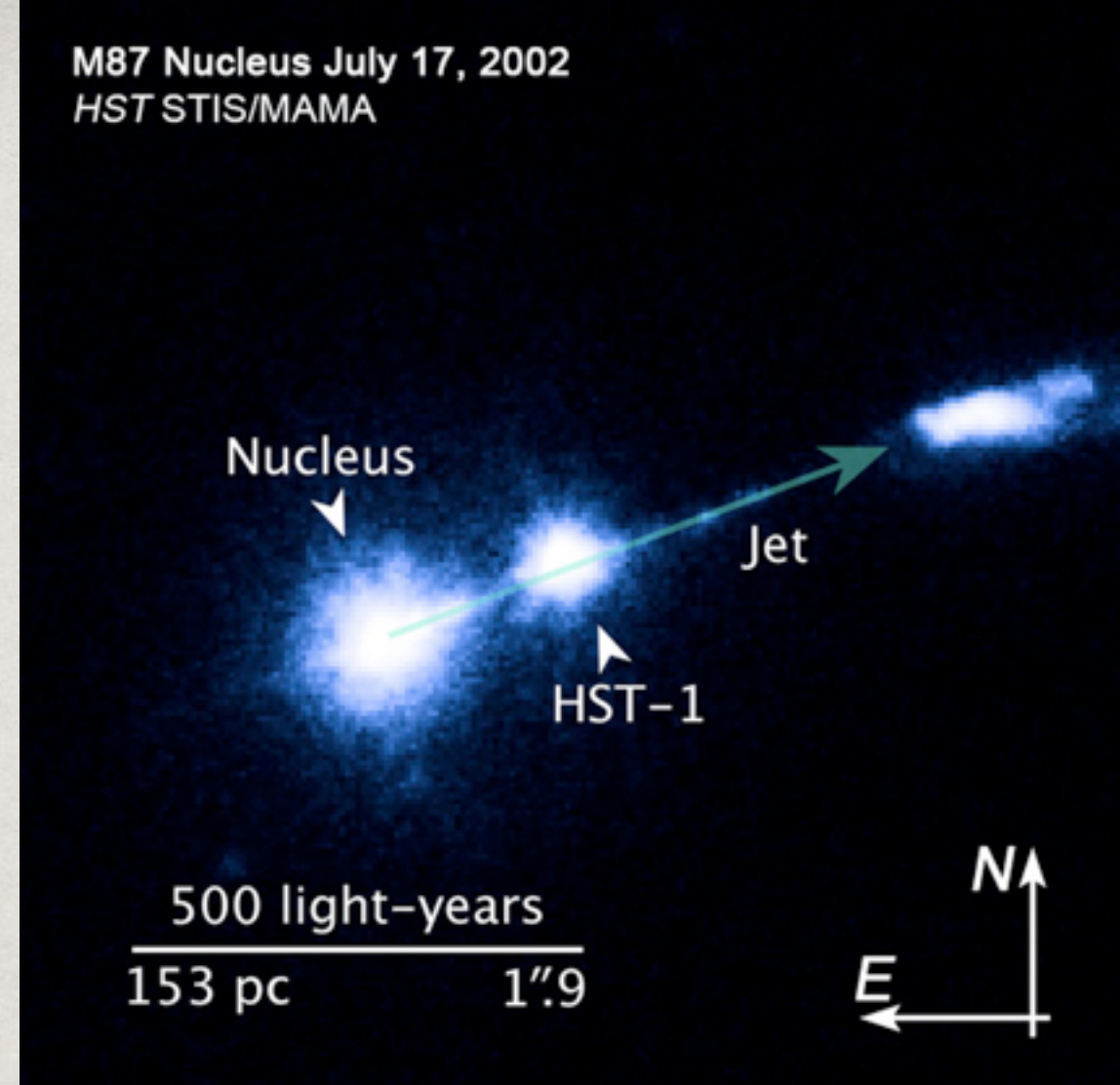
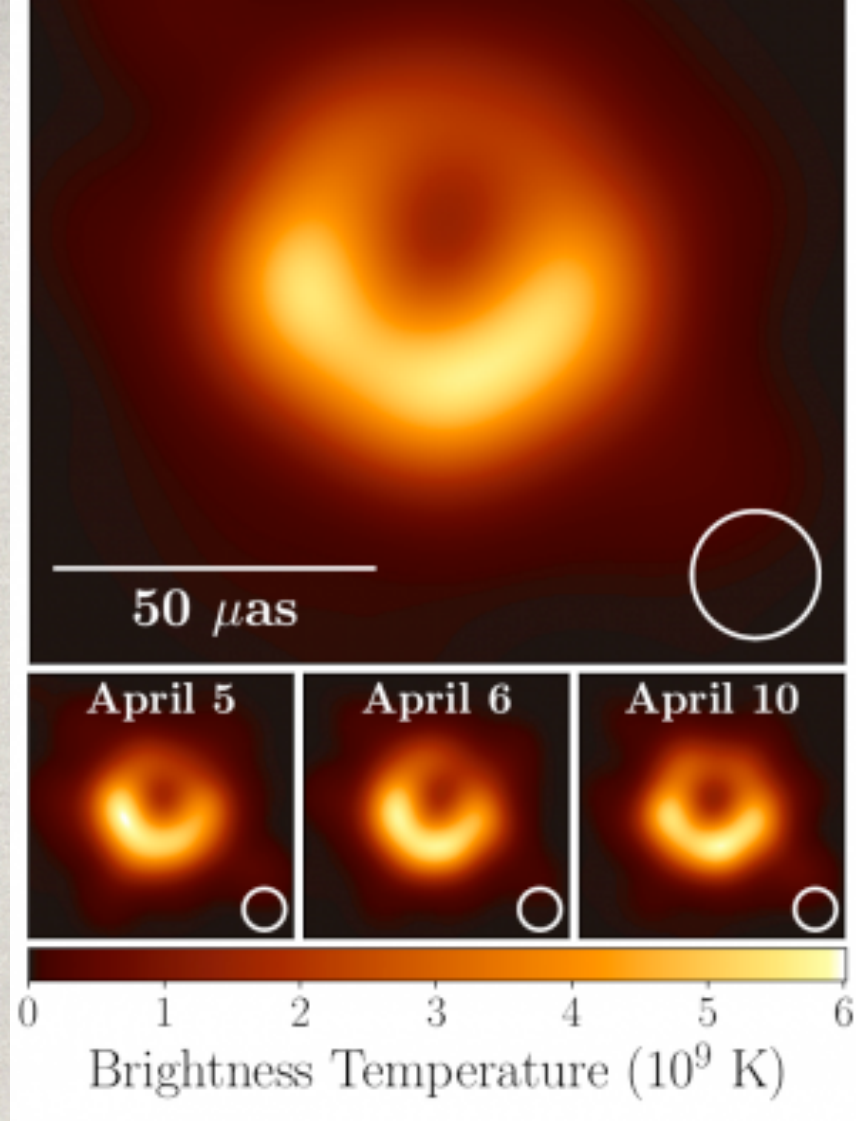
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Conservative
10% error

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Plan: to discuss strong light bending

1) Paradigm: Kerr black holes

2) Non-Kerr (but reasonable) black holes

3) (Generic) horizonless ultracompact compact objects

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Kerr is not just the unique theoretical vacuum model; it is a good model:

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Three broad theoretical criteria for a good model of compact objects:

1) Appear in a well motivated and consistent physical model;

Kerr: General Relativity

2) Have a dynamical formation mechanism;

Kerr: gravitational collapse

3) Be (sufficiently) stable.

Kerr: mode stability established (B. F. Whiting, J. Math. Phys. 30 (1989) 1301)

Check
list!

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Check
list!

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Crucially, moreover, it must give the right phenomenology:

1) all electromagnetic observables
(X-ray spectrum, shadows, QPOs, star orbits,...);

2) correct Gravitational wave templates

*No clear
tension between
observations and
the Kerr model*

Black holes beyond Kerr: two “reasonable” examples

Black holes beyond Kerr: two “reasonable” examples

a) In General Relativity, beyond the SM

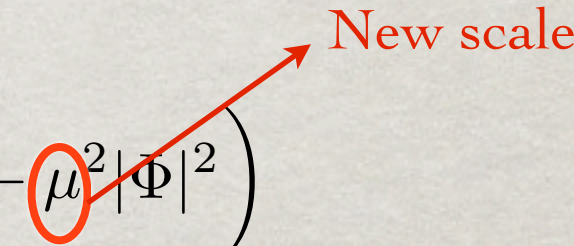
Massive-complex-scalar-vacuum:

$$\mathcal{S} = \frac{1}{4\pi} \int d^4x \sqrt{-g} \left(\frac{R}{4} - \nabla_\alpha \Phi^* \nabla^\alpha \Phi - \mu^2 |\Phi|^2 \right)$$

Black holes beyond Kerr: two “reasonable” examples

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Massive-complex-scalar-vacuum:

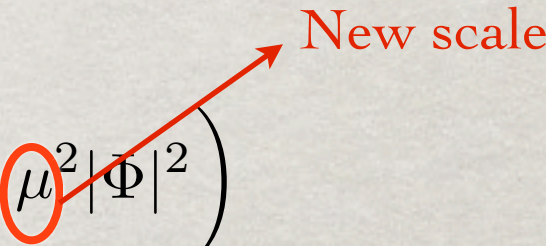
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New scale

Black holes beyond Kerr: two “reasonable” examples

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b) Beyond General Relativity

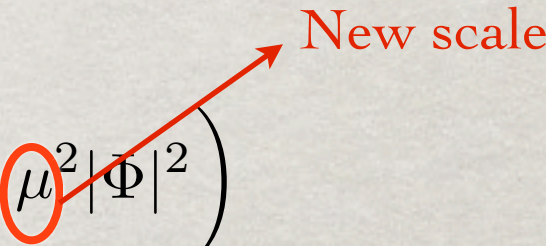
Extended scalar-tensor Gauss-Bonnet:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\partial_\mu \phi \partial^\mu \phi + \lambda^2 f(\phi) \{ R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \} \right],$$

Black holes beyond Kerr: two “reasonable” examples

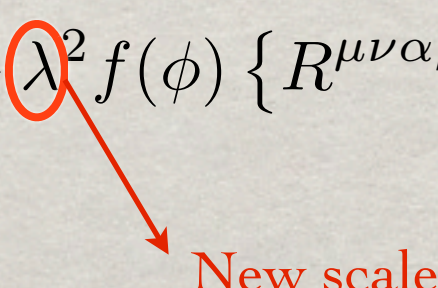
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b) Beyond General Relativity

Extended scalar-tensor Gauss-Bonnet:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\partial_\mu \phi \partial^\mu \phi + \lambda^2 f(\phi) \{ R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \} \right],$$


a) In General Relativity

Massive-complex-scalar-vacuum:

$$\mathcal{S} = \frac{1}{4\pi} \int d^4x \sqrt{-g} \left(\frac{R}{4} - \nabla_\alpha \Phi^* \nabla^\alpha \Phi - \mu^2 |\Phi|^2 \right)$$

Black Holes with synchronised hair

CH and Radu, PRL112(2014)221101

Existence proof

Chodosh and Shlapentokh-Rothman,
CMP356(2017)1155

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CMP356(2017)1155

1) Appear in a well motivated and consistent physical model;

General Relativity minimally coupled to massive bosonic fields

2) Have a dynamical formation mechanism;

Superradiance instability of Kerr

(East and Pretorius, PRL119(2017)041101,

CH, Radu, Phys. Rev. Lett. 119 (2017) 261101, Dolan, Physics10(2017)83)

3) Be (sufficiently) stable.

Effective stability against superradiance in some range of masses and couplings

(Ganchev and Santos PRL 120 (2018) 171101; Degollado, CH, Radu PLB 781 (2018) 651)

Check
list!

Select
a scale

In the space of solutions,
this model allows for large
deviations from Kerr...

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Cunha, CH, Radu, Runarsson,
Phys. Rev. Lett. 115 (2015) 211102

Infant Stars in Small Magellanic cloud (HST)



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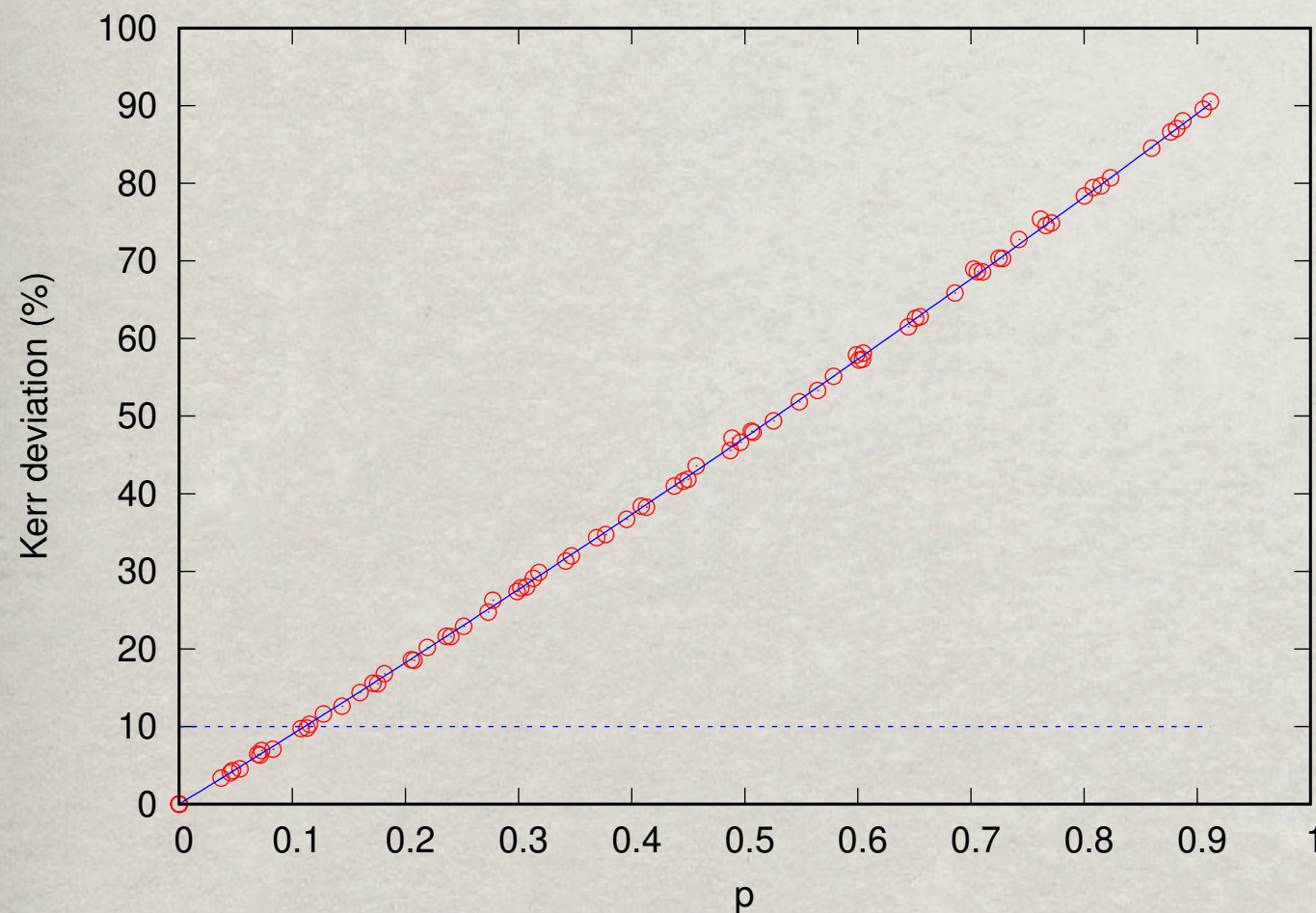
Kerr



Comparable hairy

But in the dynamically most interesting region,
differences are not large...

Degollado, CH, Radu PLB 781 (2018) 651

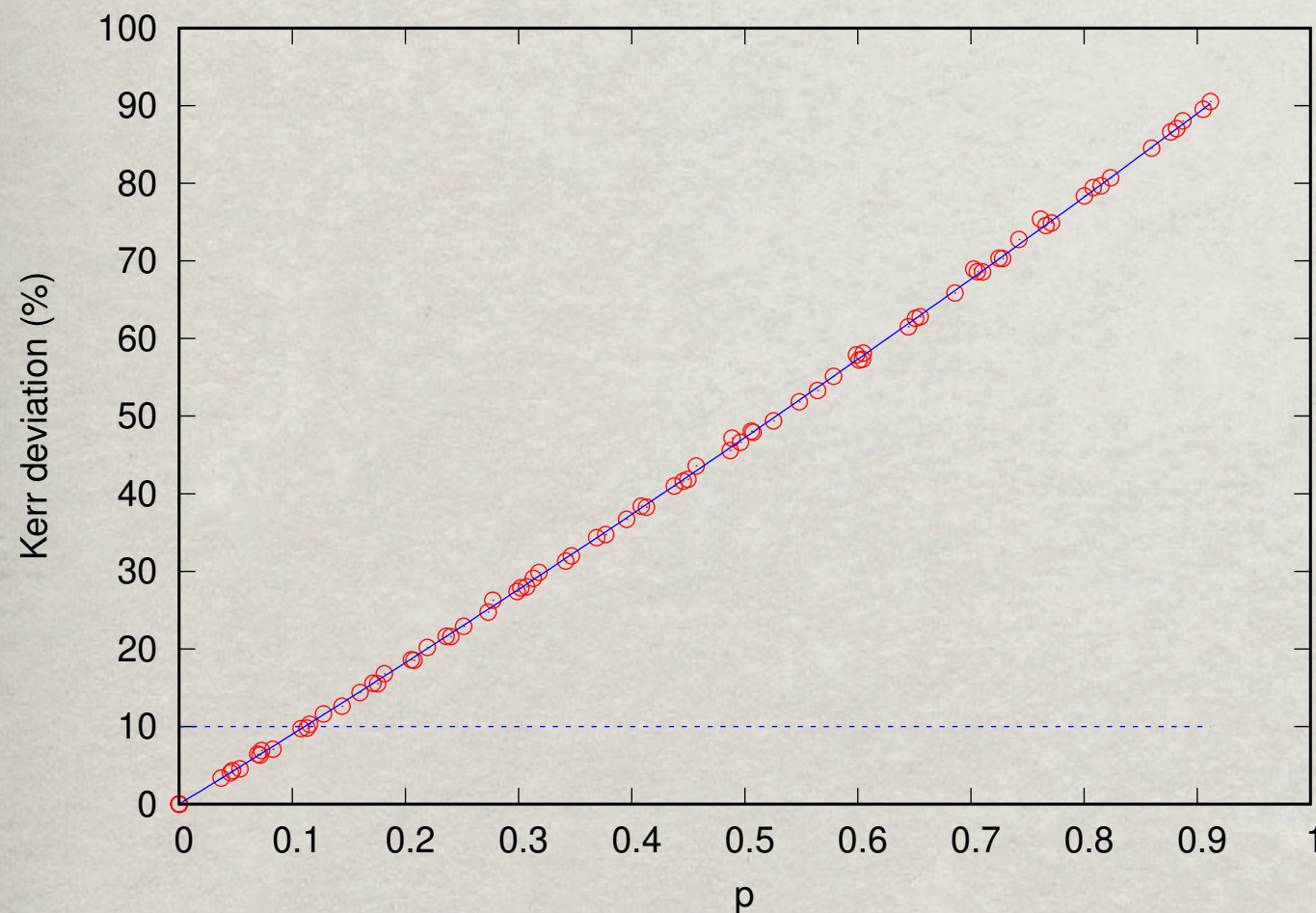


Constraint:
fraction of energy in the
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Cunha, CH, Radu,
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Constraint:
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Cunha, CH, Radu,
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Dynamically viable region of this non-Kerr black holes
is within error bars of M87 observations.

b) Beyond General Relativity

One illustrative example: Einstein-dilaton-Gauss-Bonnet
(arises in String Theory, second order equations of motion, etc...)

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial_\mu \phi)^2 + \alpha e^{-\gamma\phi} R_{\text{GB}}^2 \right],$$

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Schwarzschild/Kerr not solutions - new black holes which are stable in some regime

P. Kanti, N. E. Mavromatos, J. Rizos, K. Tamvakis and E. Winstanley, Phys. Rev. D 54 (1996) 5049; Phys. Rev. D 57 (1998) 6255;
P. Kanti, B. Kleihaus and J. Kunz, Phys. Rev. Lett. 107 (2011) 271101

New qualitative features (minimal black hole size);

Phenomenology:

No large deviations from Kerr occur; e.g. shadows

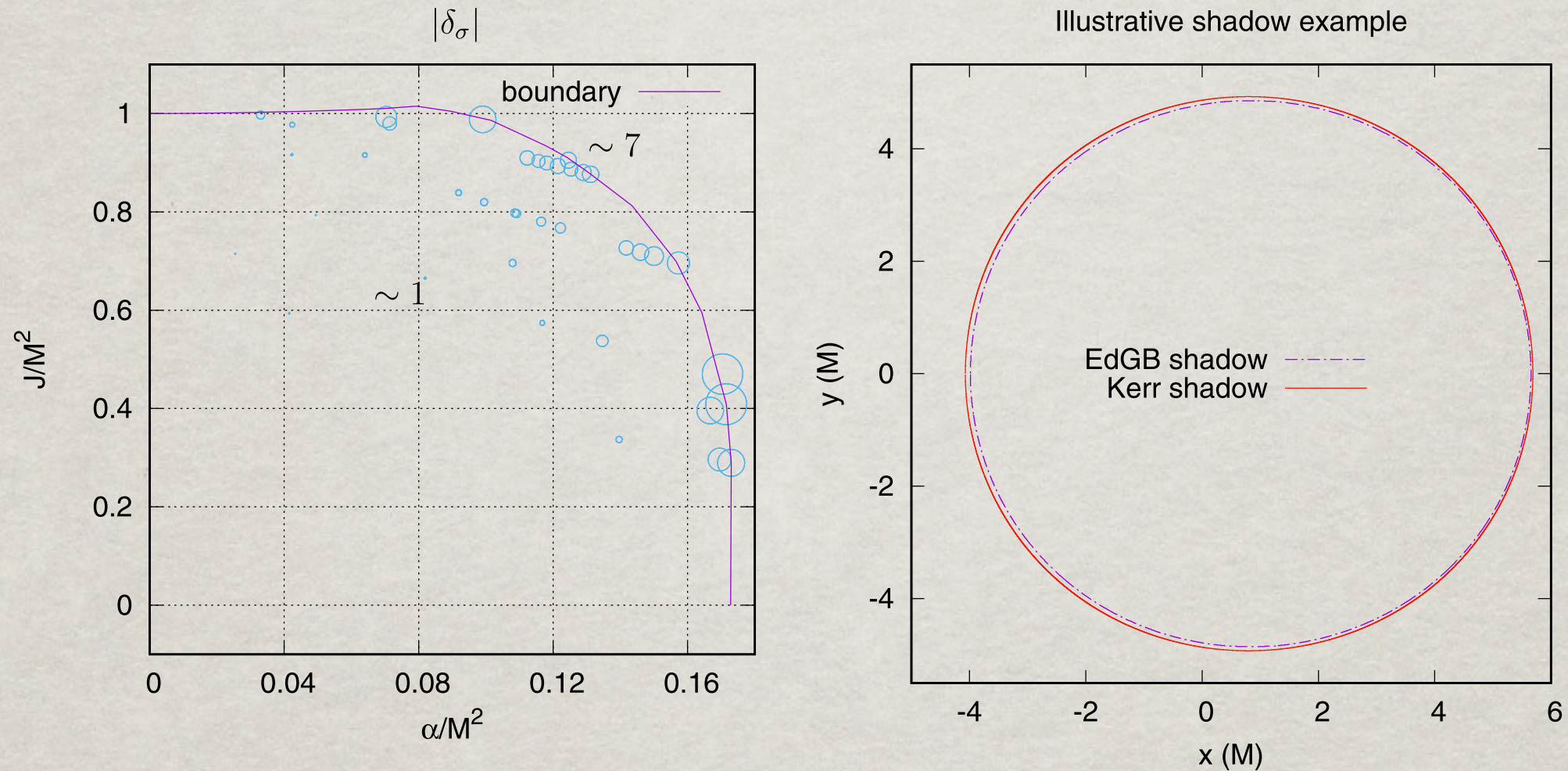


Fig. 4. (Left) Representation of $|\delta_\sigma|$ for EdGB solutions with $\gamma = 1$, in a α/M^2 vs. J/M^2 diagram. Each circle radius is proportional to the quantity represented, with some values also included for reference. All the values of δ_σ are negative. (Right) Depiction of the shadow edge of a EdGB BH with $\gamma = 1$ and $(\alpha/M^2, J/M^2) \simeq (0.172, 0.41)$, yielding $\bar{r} \simeq 4.85$, $\sigma = 0.3$, $x_C = 0.84$; the radial deviation δ_r with respect to the comparable Kerr case is $\simeq -1.35\%$. The observer is at a perimetral radius $15M$.

The case of Einstein-dilaton-Gauss-Bonnet:
the largest shadow deviation is (in the average radius) only \sim few %

Cunha, CH, Kunz, Kleihaus, Radu, PLB 768 (2017) 373

Allowing a more general coupling...

There are various interesting cousin models changing the scalar-curvature coupling:

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Examples:

Shift symmetric model (exhibits dynamical formation):

$$f(\phi) = \alpha\phi$$

Sotiriou and Zhou, Phys. Rev. D 90 (2014) 124063

Benkel, Sotiriou and Witek, Phys. Rev. D 94 (2016) 121503

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Models admitting vacuum GR **and** hairy black holes:

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Doneva and Yazadjiev, Phys. Rev. Lett. 120 (2018) 131103

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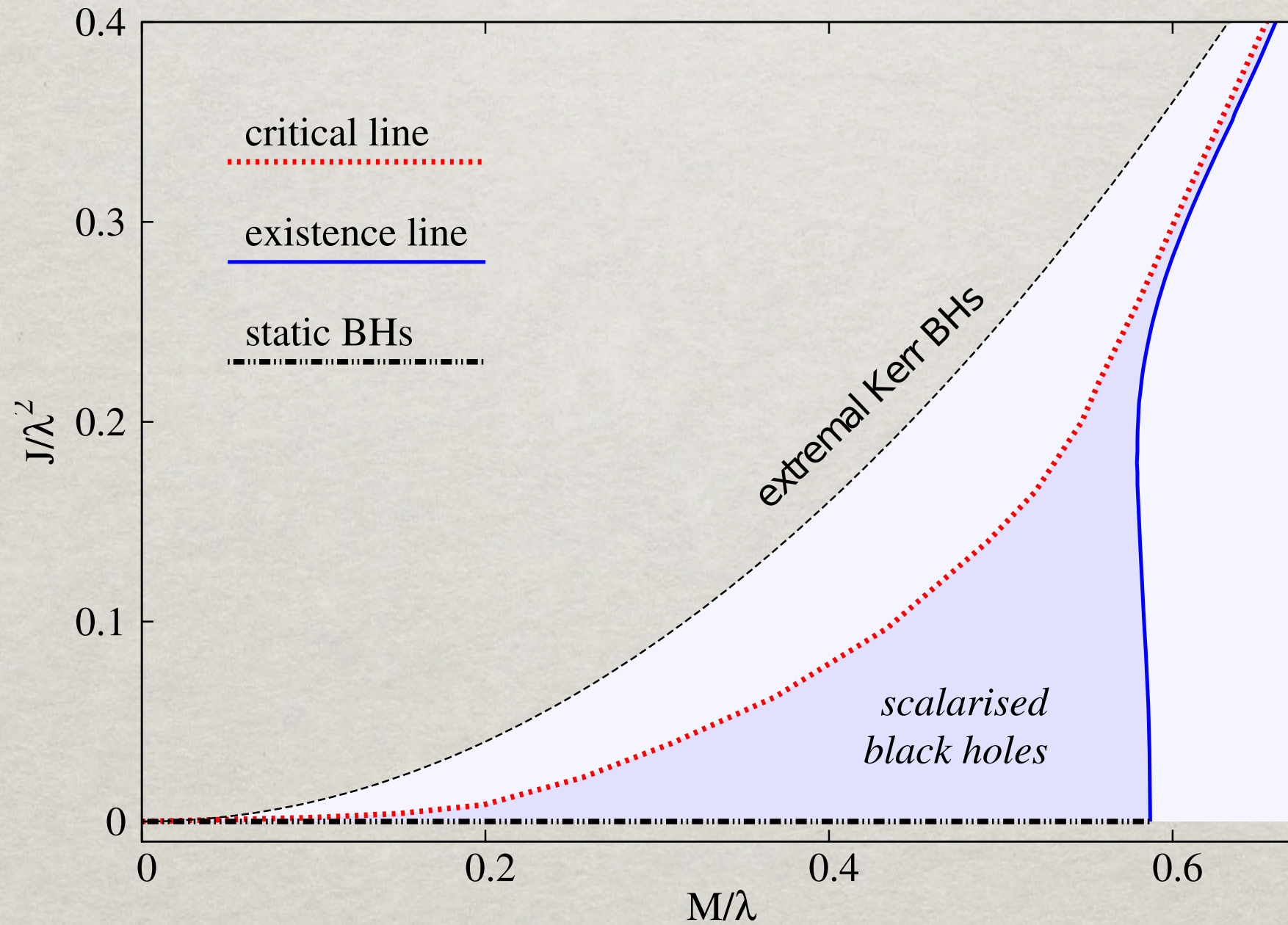
Suggests *spontaneous scalarisation* of vacuum black holes

Illustrative example: $f(\phi) = \frac{1}{2\beta}(1 - e^{-\beta\phi^2})$

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\partial_\mu \phi \partial^\mu \phi + \lambda^2 f(\phi) \left\{ R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \right\} \right],$$

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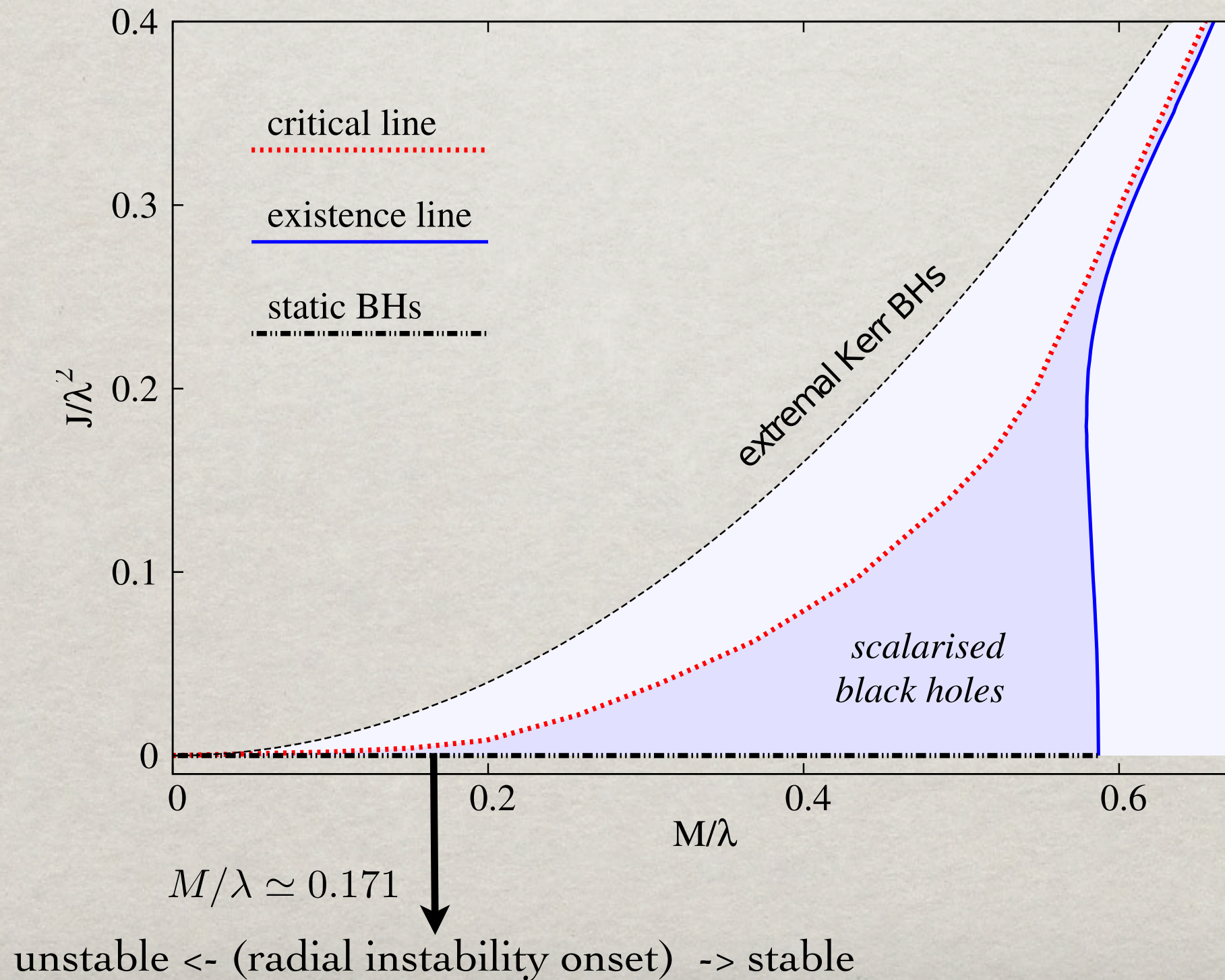
P. Cunha, C. H., E. Radu, arXiv:1904.09997

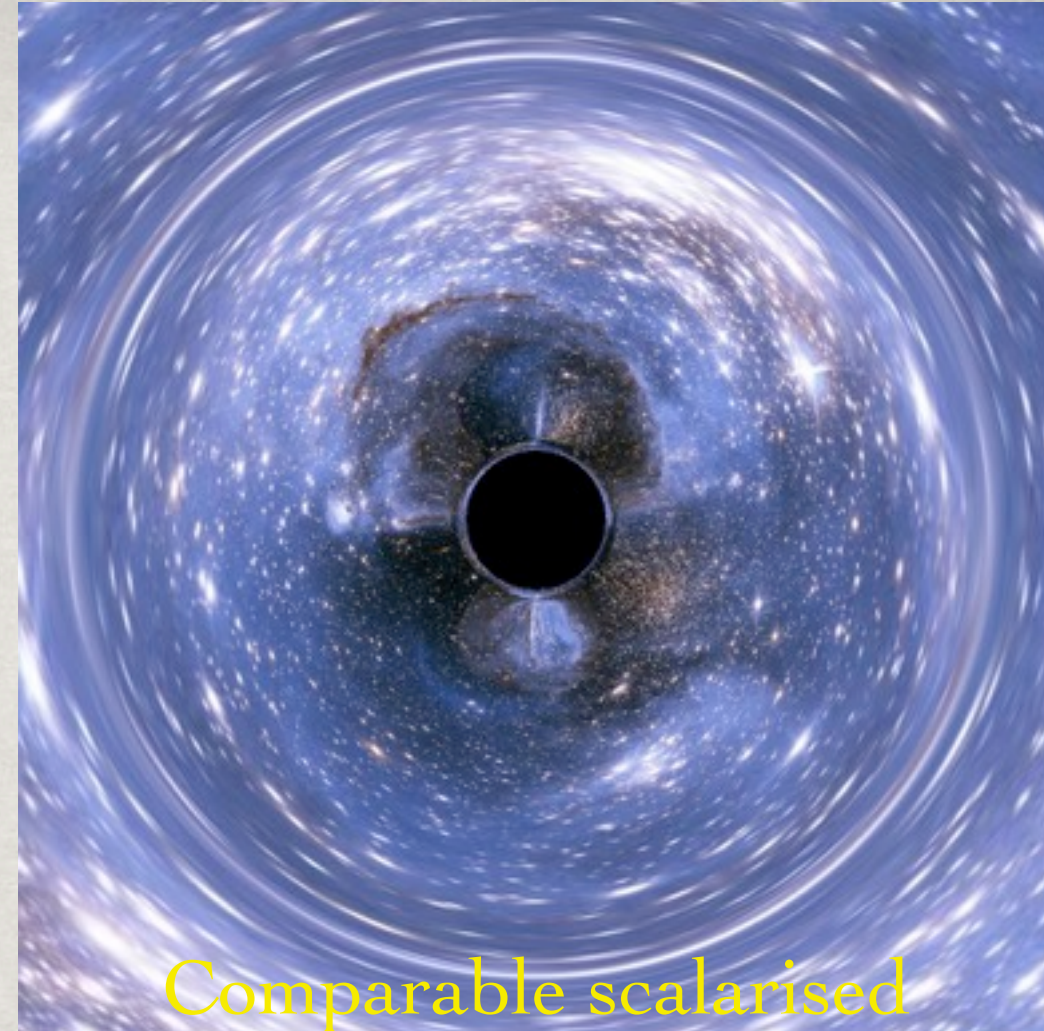


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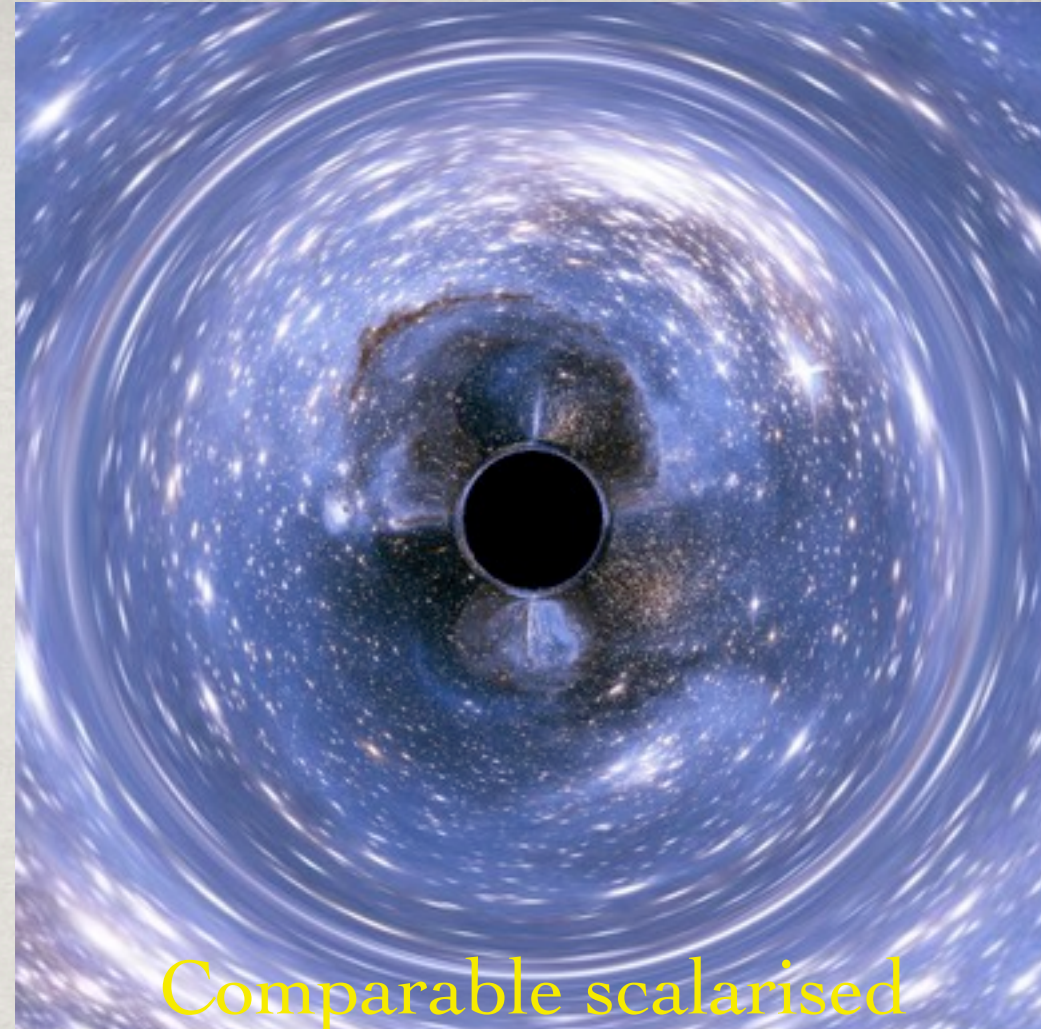
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Schwarzschild



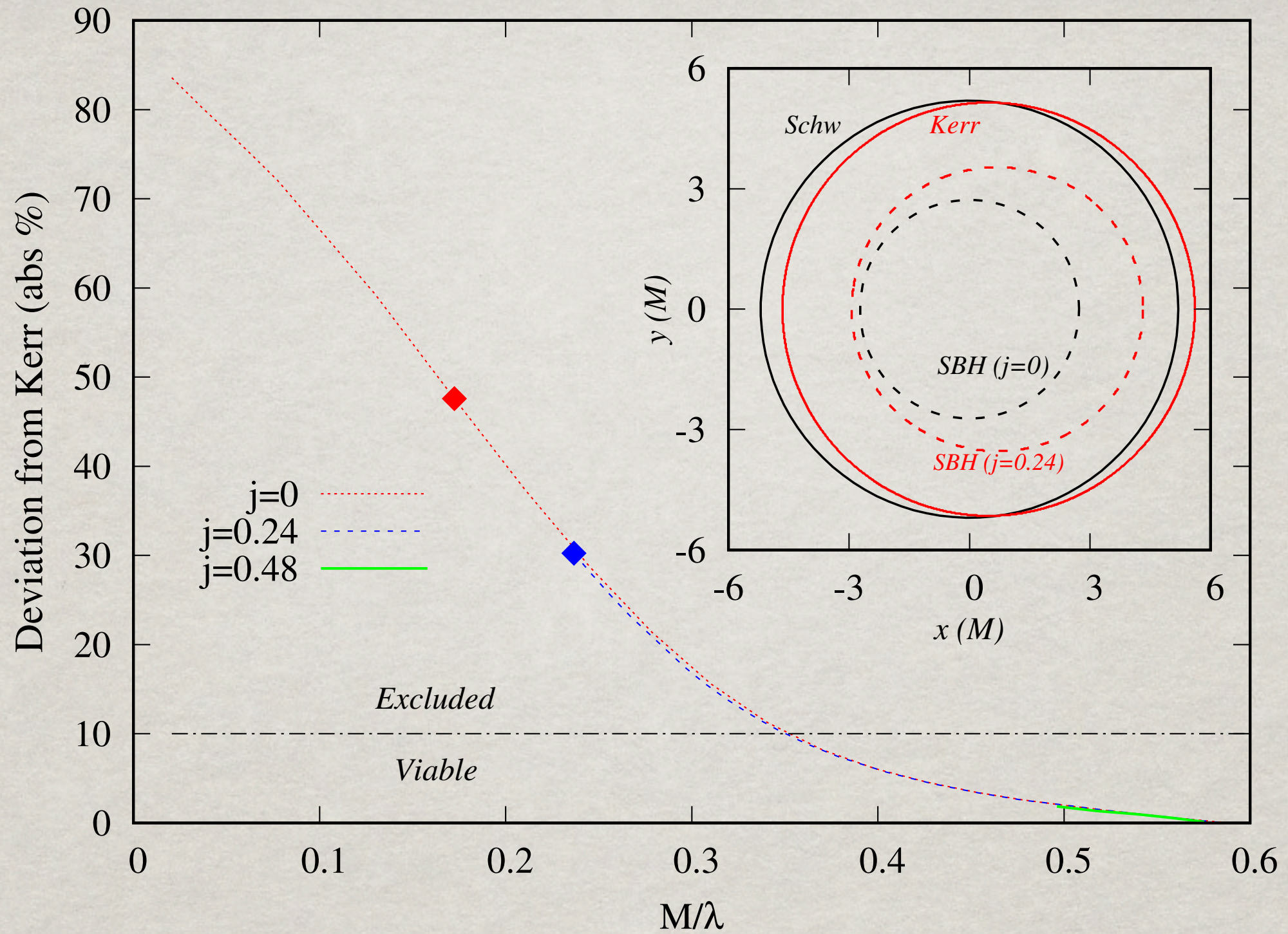
Comparable scalarised



Kerr, $j=0.24$



Comparable scalarised



If M87 black hole spin is small, yields (weak) constraint:

$$\lambda \lesssim 1.8 \times 10^{10} M_{\odot}$$

b) Beyond General Relativity

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1) Appear in a well motivated and consistent physical model;

Einstein-dilaton-Gauss-Bonnet / extended scalar tensor Gauss Bonnet

2) Have a dynamical formation mechanism;

Gravitational collapse or scalarisation

3) Be (sufficiently) stable.

Some solutions are stable

Check
list!

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Issues:

- Why stop at quadratic curvature?
 - Why a certain coupling?
- There are effective violations of energy conditions. Is it an issue?



News in “Jornal da ilha da Madeira”
and “Estado do Pará” about the 1919 expeditions

Plan: to discuss strong light bending

1) Paradigm: Kerr black holes

2) Non-Kerr (but reasonable) black holes

3) (Generic) horizonless ultracompact compact objects

4) Epilogue;

The light ring determines the initial “ringdown” of a perturbed black hole

Goebel, *Astrophys. J.* 172 (1972) L95

THE ASTROPHYSICAL JOURNAL, 172:L95–L96, 1972 March 15

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COMMENTS ON THE “VIBRATIONS” OF A BLACK HOLE

C. J. GOEBEL

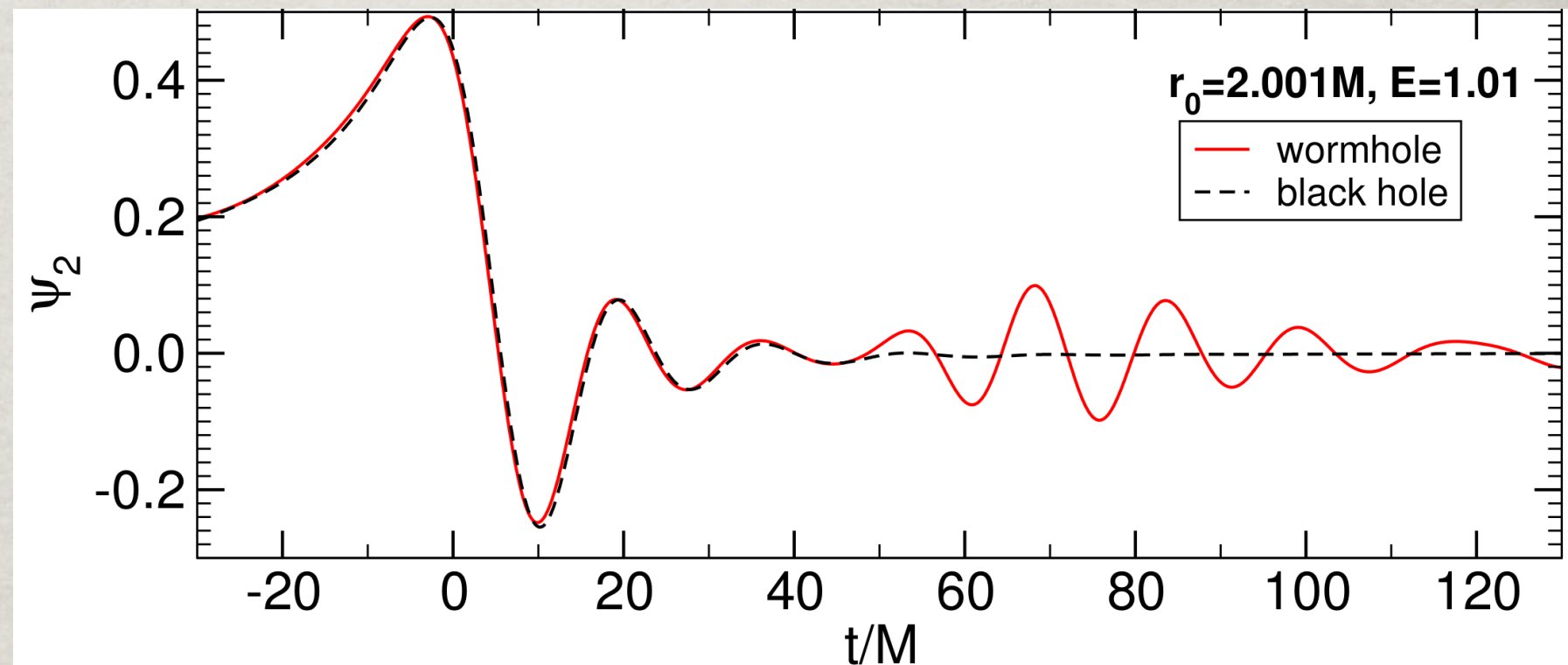
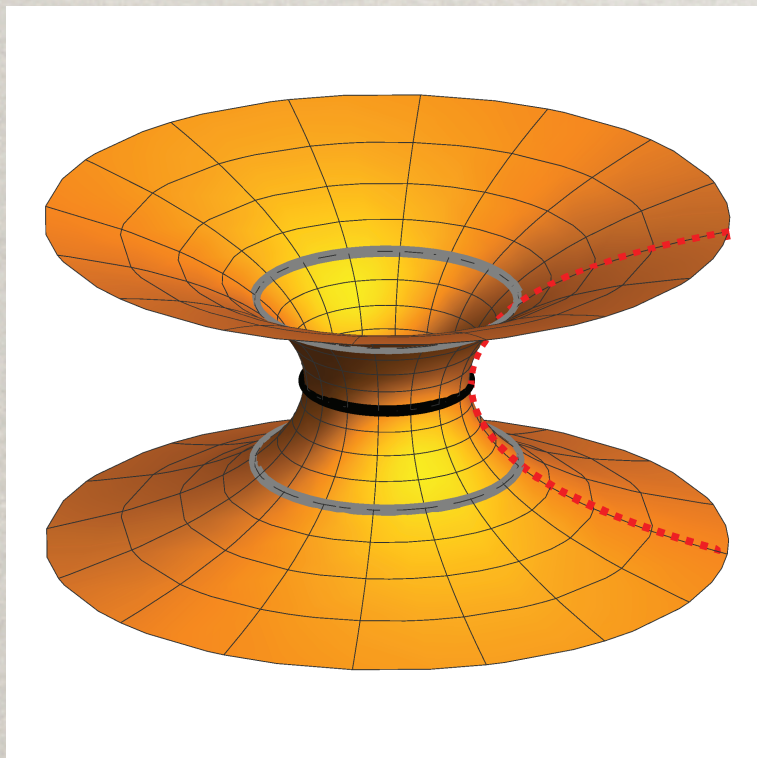
University of Wisconsin, Physics Department, Madison

Received 1972 January 4; revised 1972 January 27

ABSTRACT

It is shown that the “vibrations of a black hole” of Press are gravitational waves in spiral orbits close to the well-known unstable circular orbit at $r = 3M$. The corresponding “vibrations” of a spinning black hole are discussed. It is emphasized that these “vibrations” provide, not a source, but only a temporary storage, of high-frequency gravitational radiation.

So, a hypothetical horizonless ultra compact object (UCO)
(i.e. with a similar light ring)
could vibrate similarly, initially...



Cardoso, Franzin, Pani, PRL 117 (2016) 089902

It turns out that for UCOs,
in a generic classical dynamical formation scenario,
this light ring is not alone...

A theorem on light rings for UCOs

PRL **119**, 251102 (2017)

PHYSICAL REVIEW LETTERS

week ending
22 DECEMBER 2017

Light-Ring Stability for Ultracompact Objects

Pedro V. P. Cunha,^{1,2} Emanuele Berti,^{3,2} and Carlos A. R. Herdeiro¹

¹*Departamento de Física da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*

²*CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa,
Avenida Rovisco Pais 1, 1049 Lisboa, Portugal*

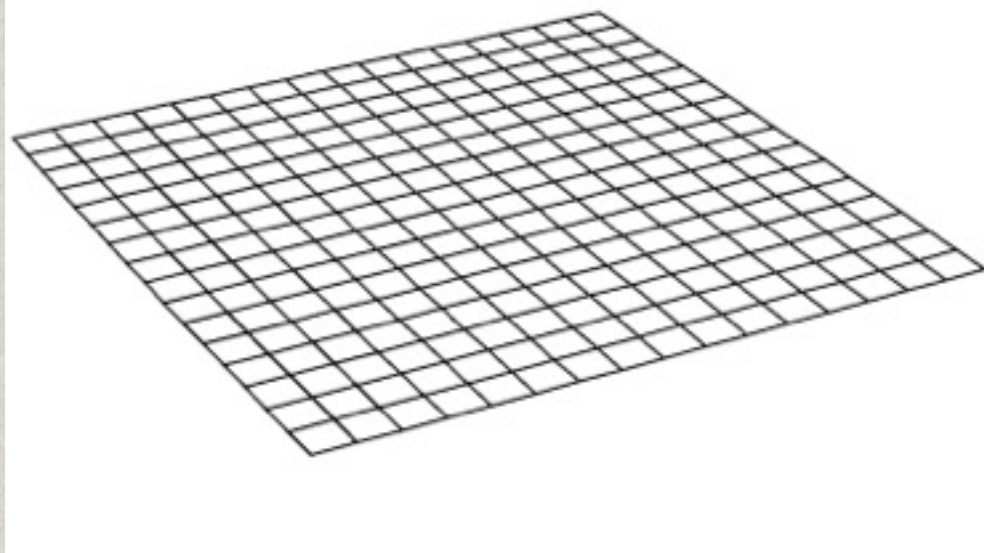
³*Department of Physics and Astronomy, The University of Mississippi, University, Mississippi 38677, USA*
(Received 3 August 2017; revised manuscript received 18 October 2017; published 18 December 2017)

We prove the following theorem: axisymmetric, stationary solutions of the Einstein field equations formed from classical gravitational collapse of matter obeying the null energy condition, that are everywhere smooth and ultracompact (i.e., they have a light ring) must have at least *two* light rings, and one of them is *stable*. It has been argued that stable light rings generally lead to nonlinear spacetime instabilities. Our result implies that smooth, physically and dynamically reasonable ultracompact objects are not viable as observational alternatives to black holes whenever these instabilities occur on astrophysically short time scales. The proof of the theorem has two parts: (i) We show that light rings always come in pairs, one being a saddle point and the other a local extremum of an effective potential. This result follows from a topological argument based on the Brouwer degree of a continuous map, with no assumptions on the spacetime dynamics, and, hence, it is applicable to any metric gravity theory where photons follow null geodesics. (ii) Assuming Einstein's equations, we show that the extremum is a local minimum of the potential (i.e., a stable light ring) if the energy-momentum tensor satisfies the null energy condition.

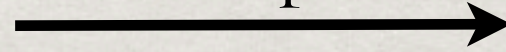
DOI: [10.1103/PhysRevLett.119.251102](https://doi.org/10.1103/PhysRevLett.119.251102)

Idea:

Start with approximately Minkowski



Incomplete
gravitational
collapse

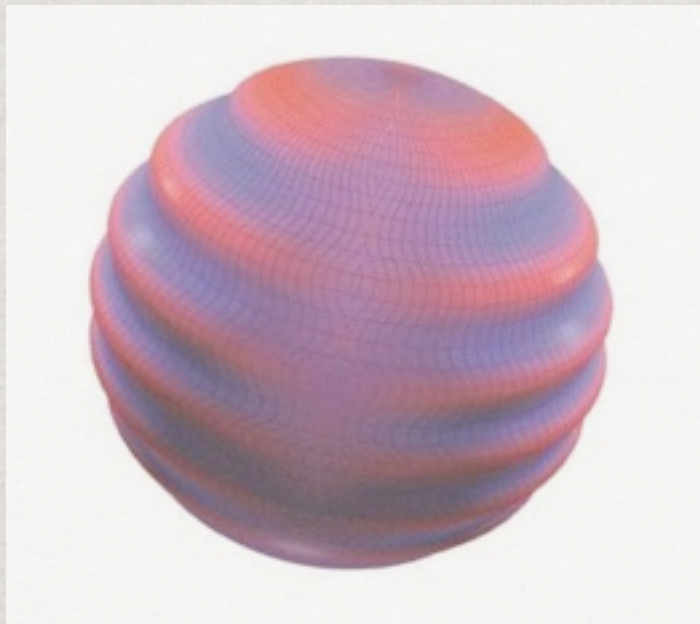


UCO as final
equilibrium state



- Starting point: approximately flat spacetime;
- UCO forms dynamically from incomplete gravitational collapse;
- End point: UCO is stationary, axi-symmetric and asymptotically flat; it has no event horizon and its metric is smooth:
- then UCO spacetime is topologically trivial, assuming causality [Geroch J.Math.Phys. 8 \(1967\) 782](#)

Null geodesic flow in UCO geometry



- determined by the Hamiltonian $\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = 0$.
- $2\mathcal{H} = (g^{ij} p_i p_j) + (g^{ab} p_a p_b), \quad i \in \{r, \theta\}, \quad a \in \{t, \varphi\}.$
 $= K + U(r, \theta).$
- Killing vectors $\partial_t, \partial_\varphi \implies E = -p_t, \quad L = p_\varphi \quad (\text{constants}).$
- $p_r = p_\theta = 0 \iff K = 0 \iff U = 0 .$

Effective potentials

- Shortcoming of $U \rightarrow$ depending on Killing parameters E, L .
- Can be factorized as $U = (L^2 g^{tt})(\sigma - H_+)(\sigma - H_-)$, $\sigma \equiv E/L$.
- Explicitly $H_{\pm}(r, \theta) = \left(-g_{t\varphi} \pm \sqrt{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}} \right) / g_{\varphi\varphi}$
- $U = 0 \iff (\sigma = H_+ \vee \sigma = H_-)$

At a LR: $\implies \boxed{\nabla H_{\pm} = 0}$ (critical point of $H_{\pm}(r, \theta)$)

- One will now associate a topological quantity w to each LR..

For Schwarzschild:

$$H_{\pm} = \pm \frac{\sqrt{1 - \frac{2M}{r}}}{r \sin \theta}$$

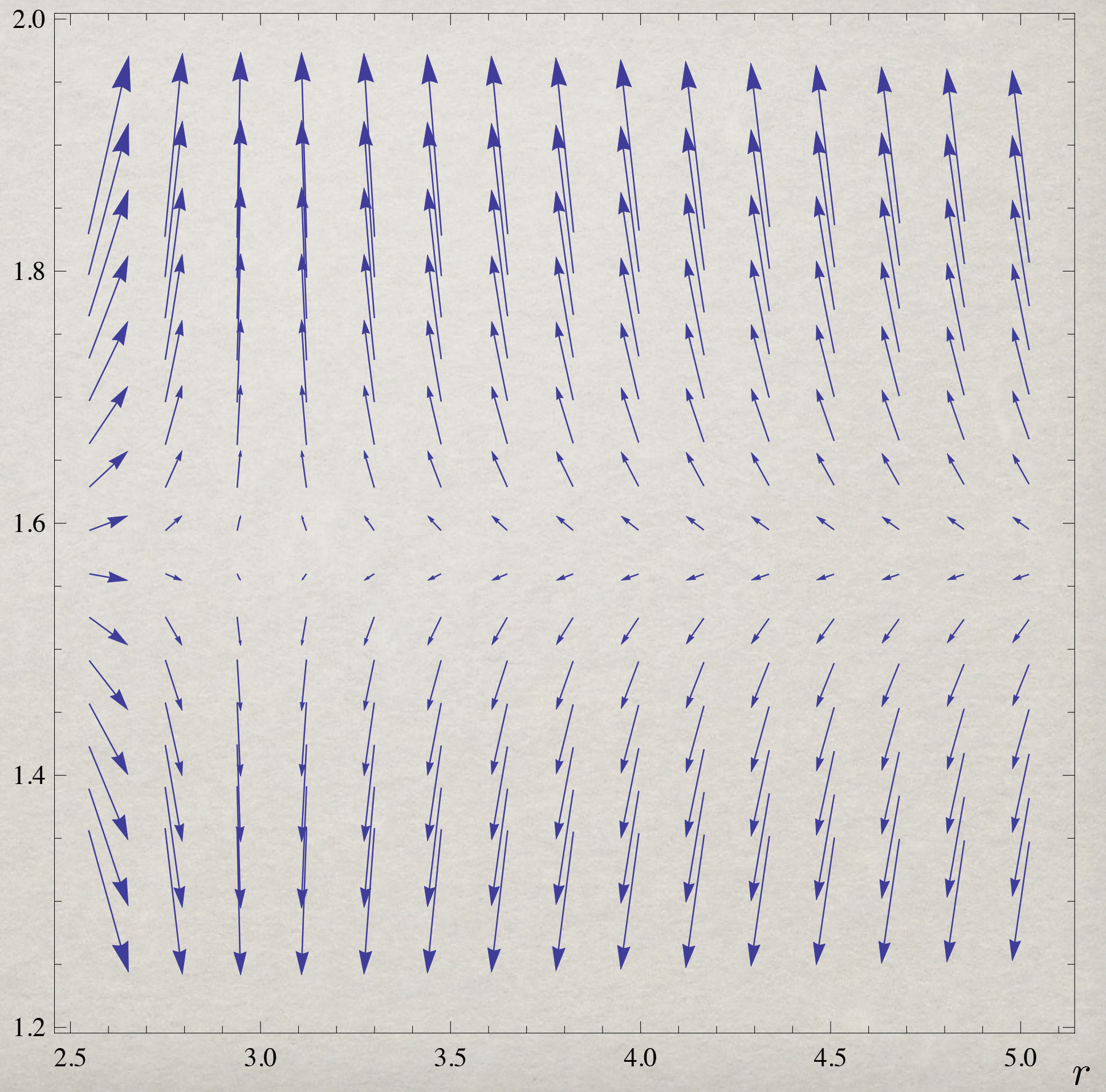
$$\mathbf{V}_{\pm} = \nabla H_{\pm}$$

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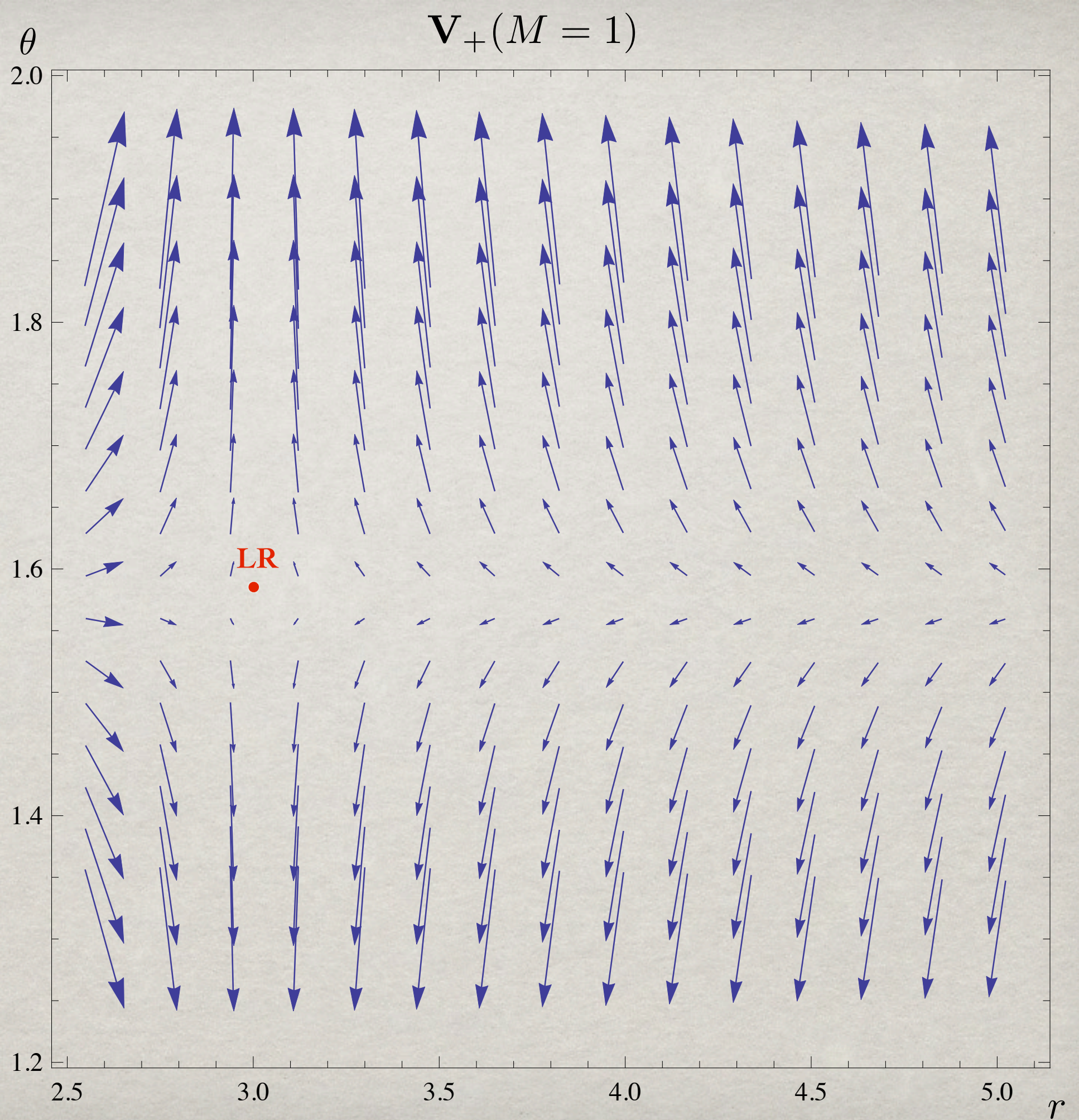
$\mathbf{V}_{+}(M = 1)$



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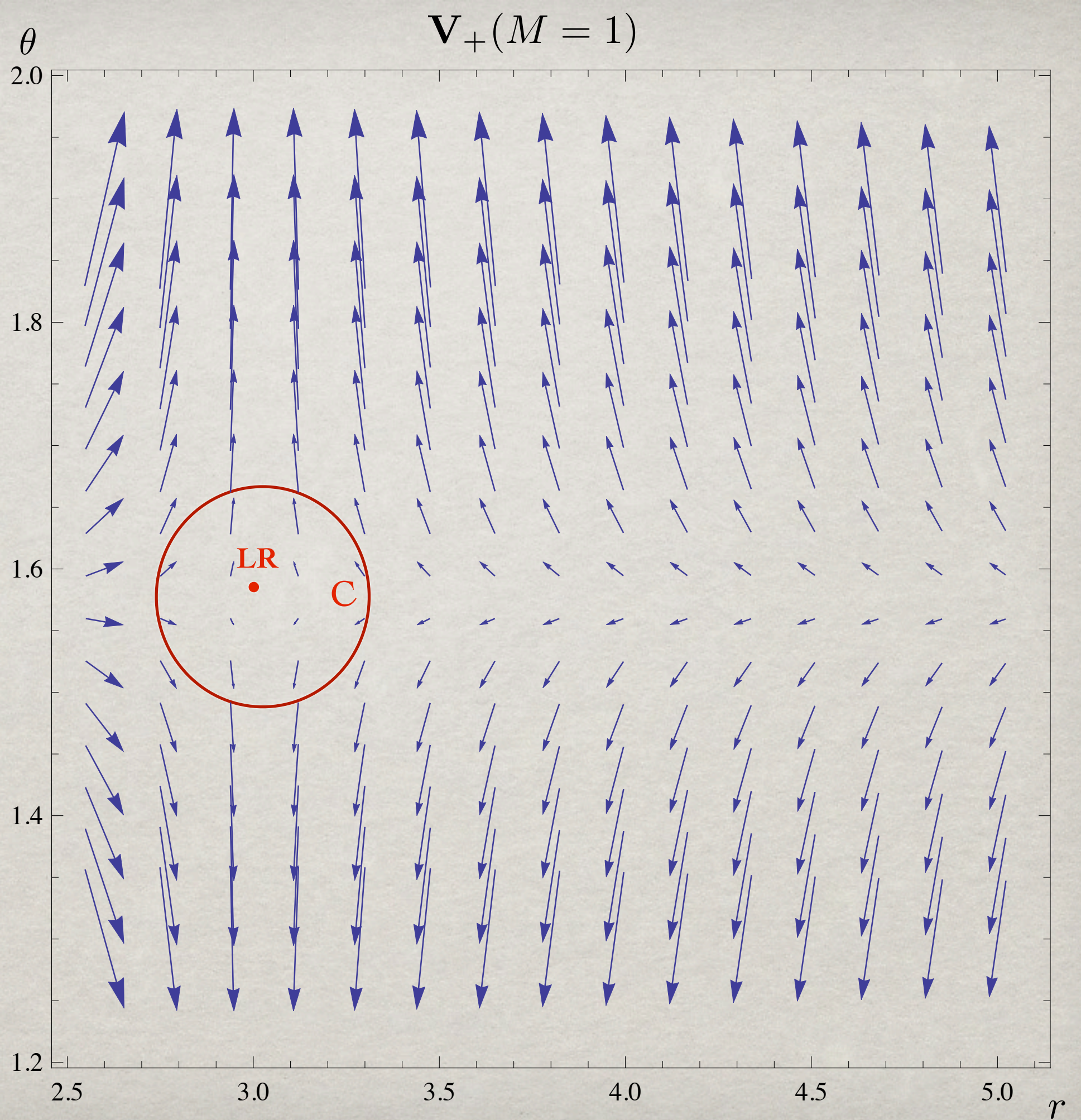
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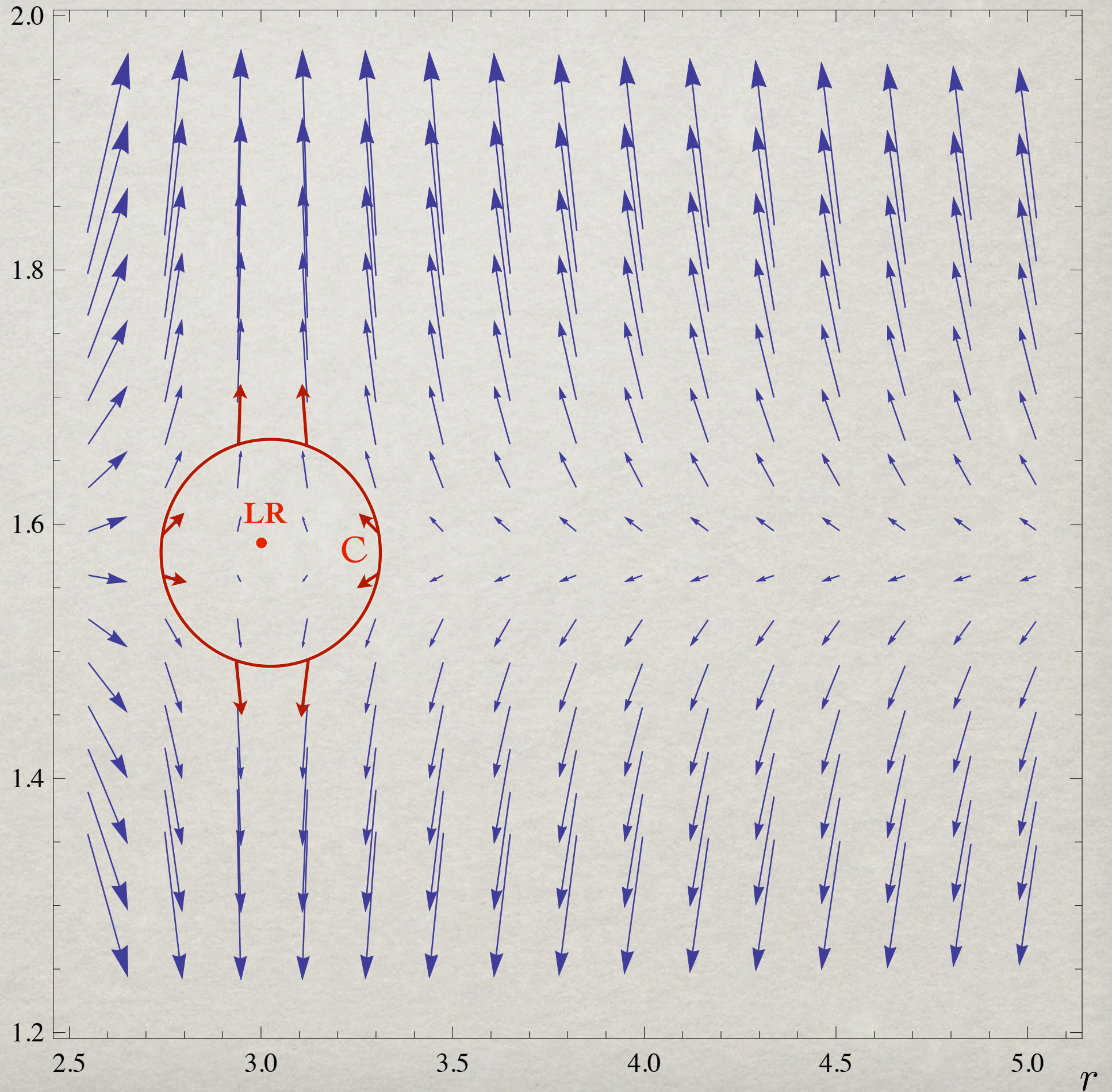


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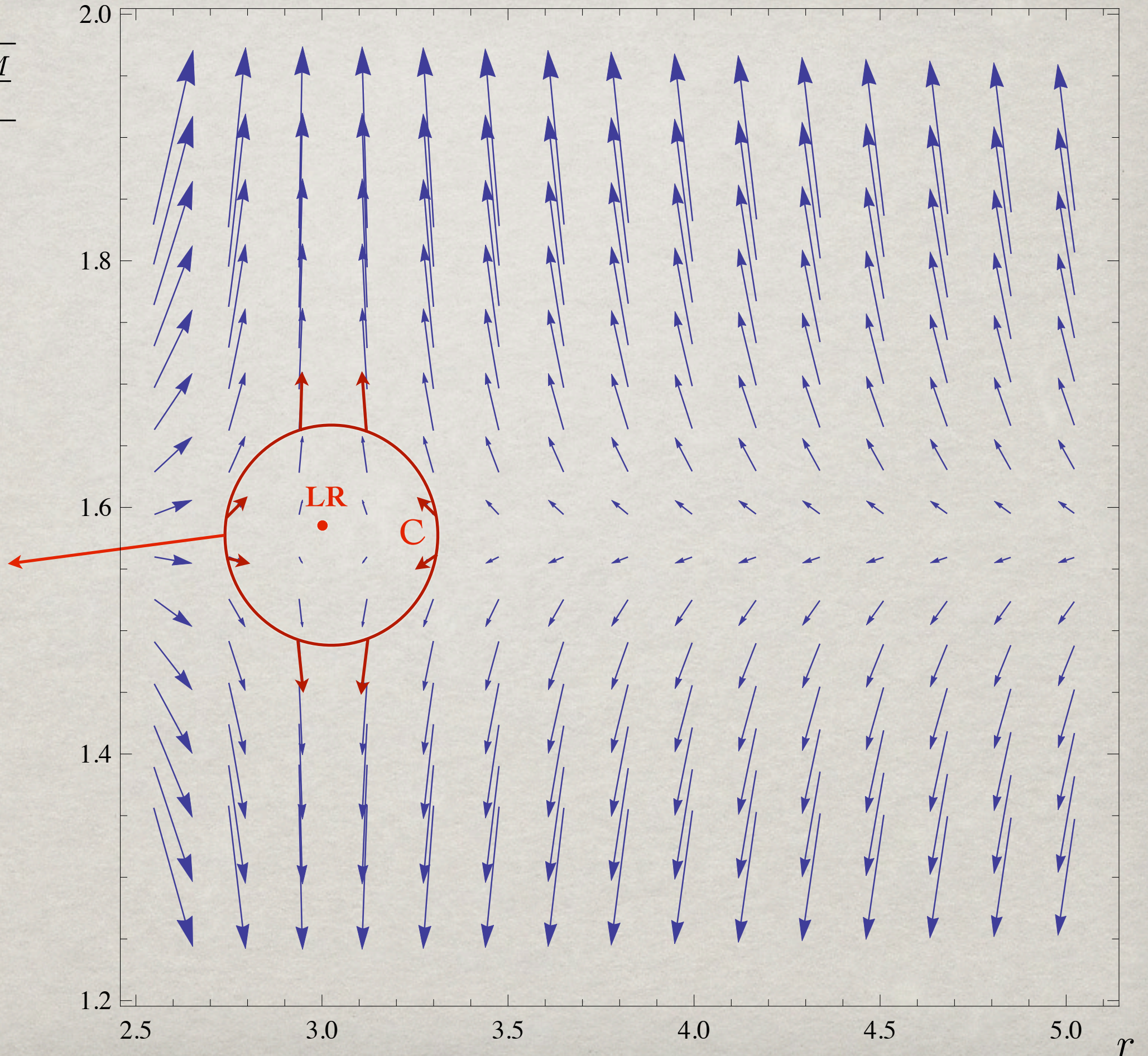
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Anti-clockwise
circulation of \mathbf{V}
around \mathbf{C} gives a
non-trivial
clockwise winding
($w=-1$)

$\mathbf{V}_{+}(M=1)$



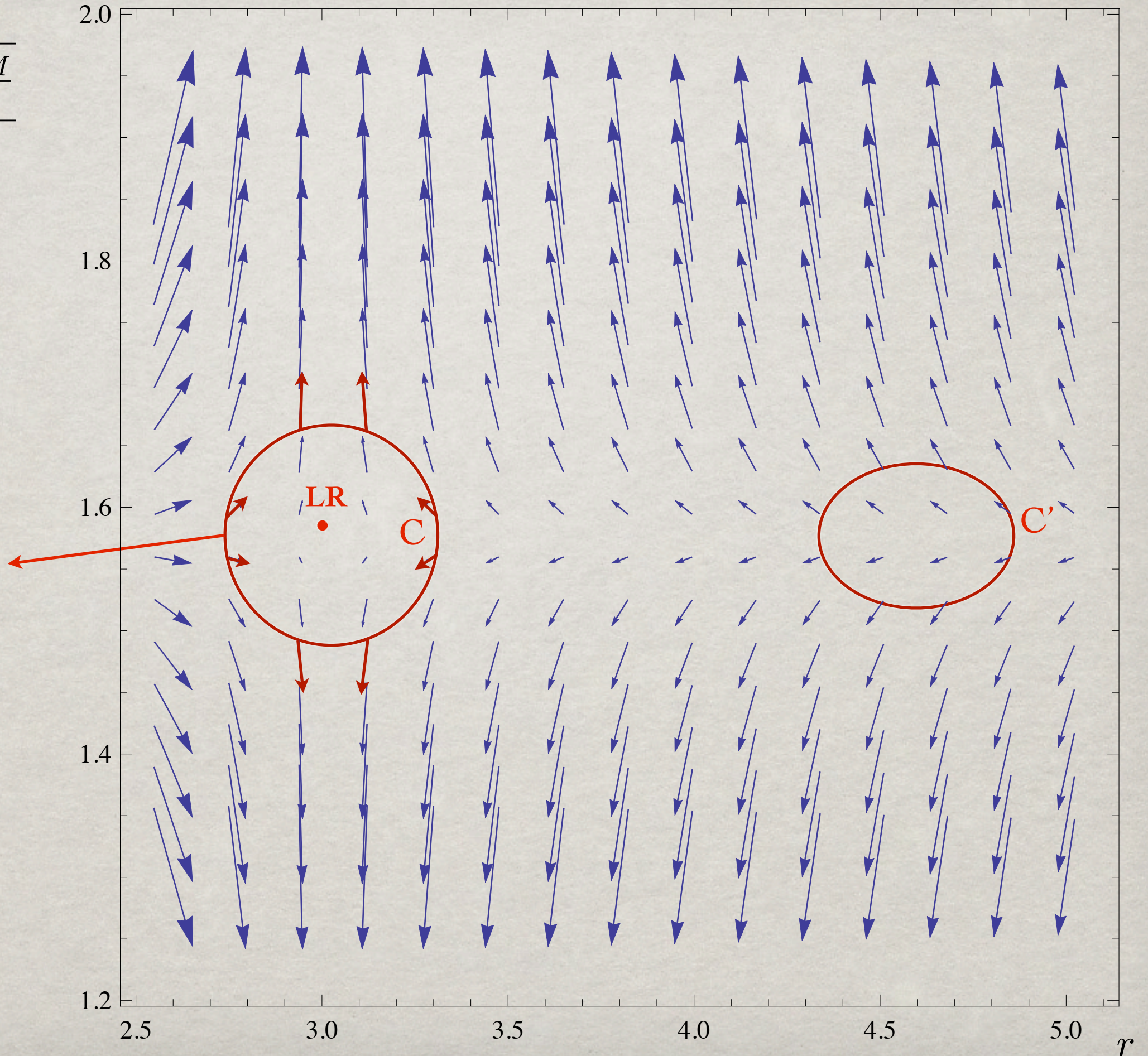
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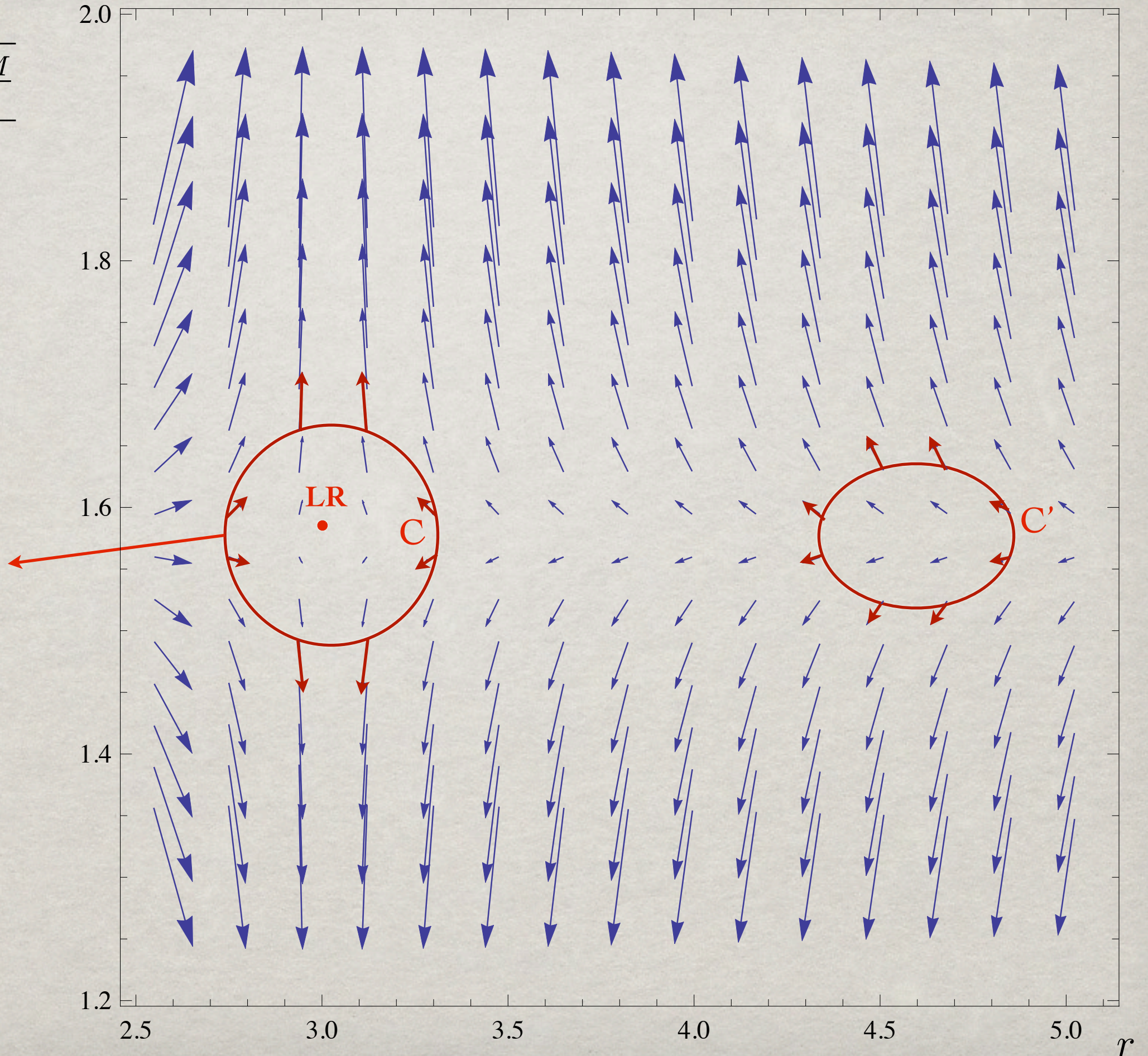
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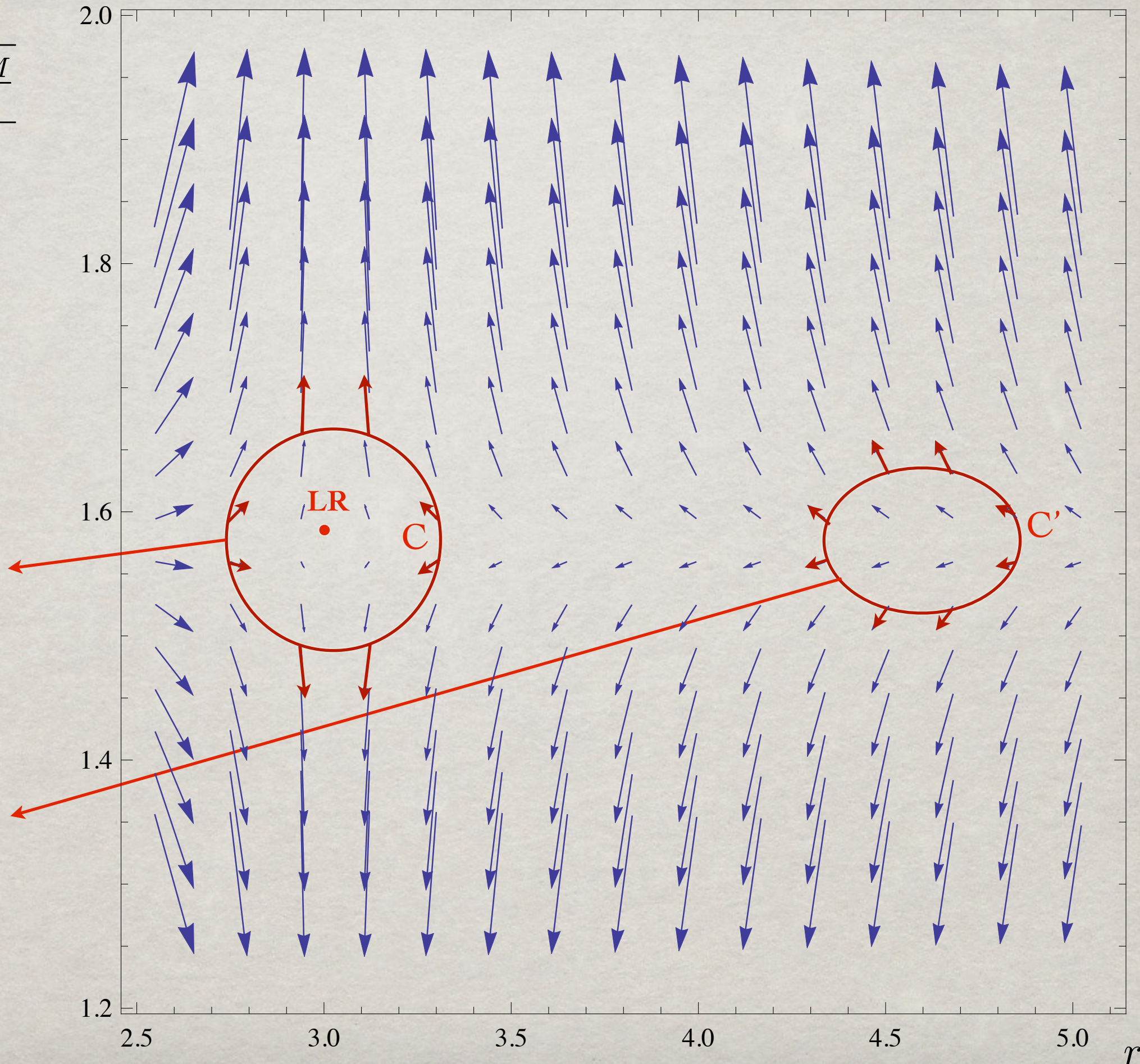
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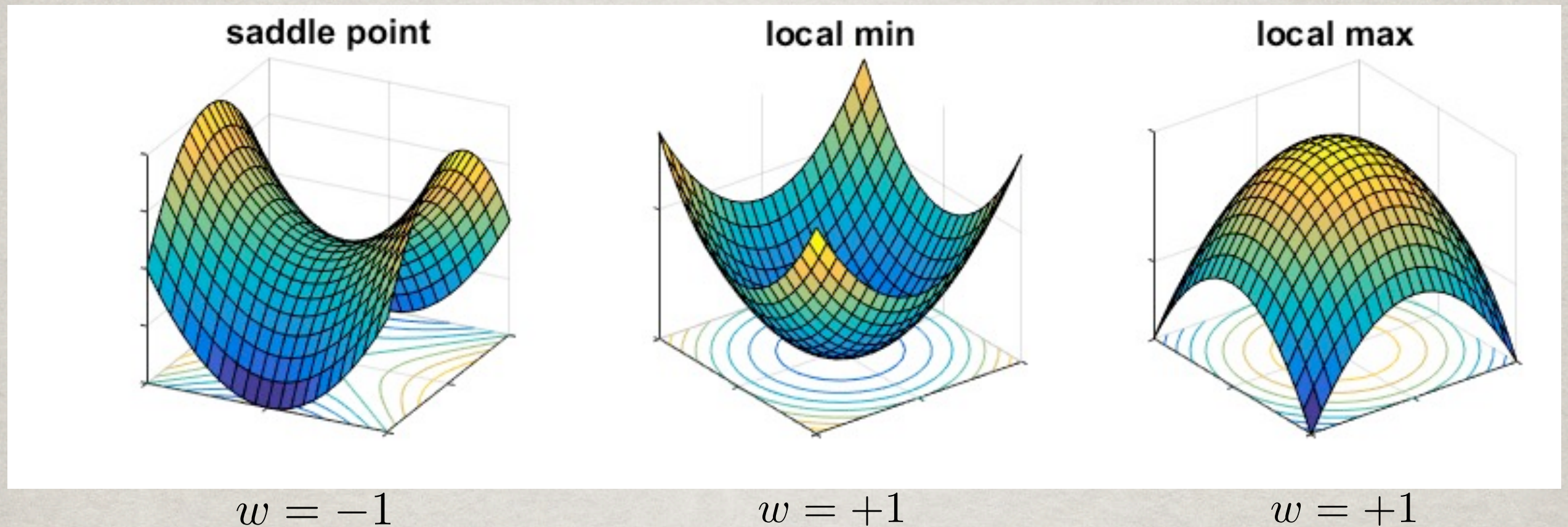
$\mathbf{V}_{+}(M = 1)$

Anti-clockwise
circulation of \mathbf{V}
around \mathbf{C} gives a
non-trivial
clockwise winding
($w=-1$)

Circulation of \mathbf{V}
around \mathbf{C}' does not
wind around
($w=0$)



Light ring types

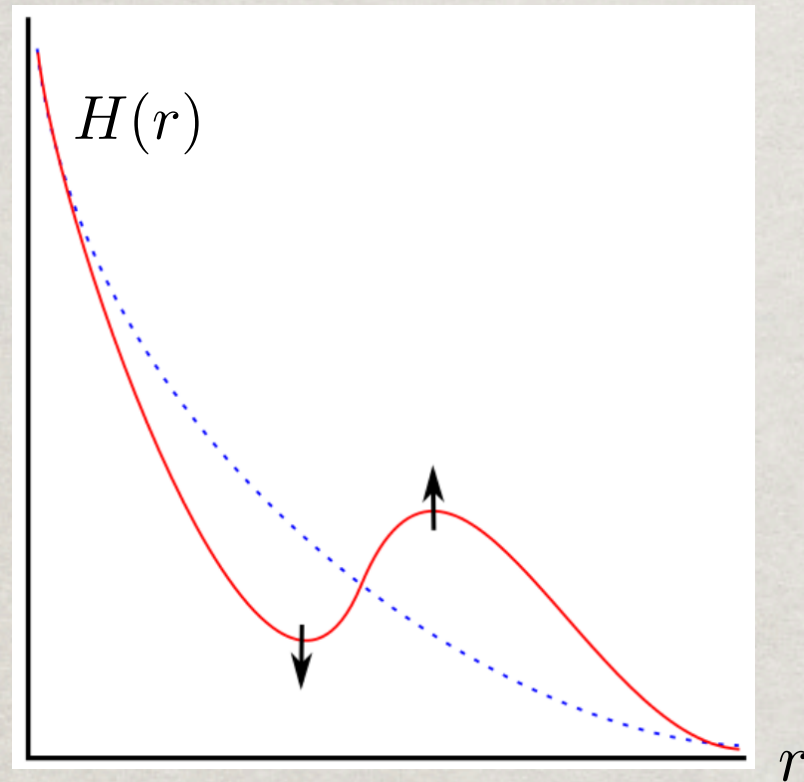


Different types of Light Rings:

- Saddle point of $H \rightarrow$ unstable LR ($w = -1$).
- Local minimum of $H \rightarrow$ stable LR ($w = +1$).
- Local maximum of $H \rightarrow$ unstable LR ($w = +1$).

Light rings are created in pairs (e.g. spherical symmetry)

- In spherical symmetry \rightarrow potential $H(r)$ is 1D.



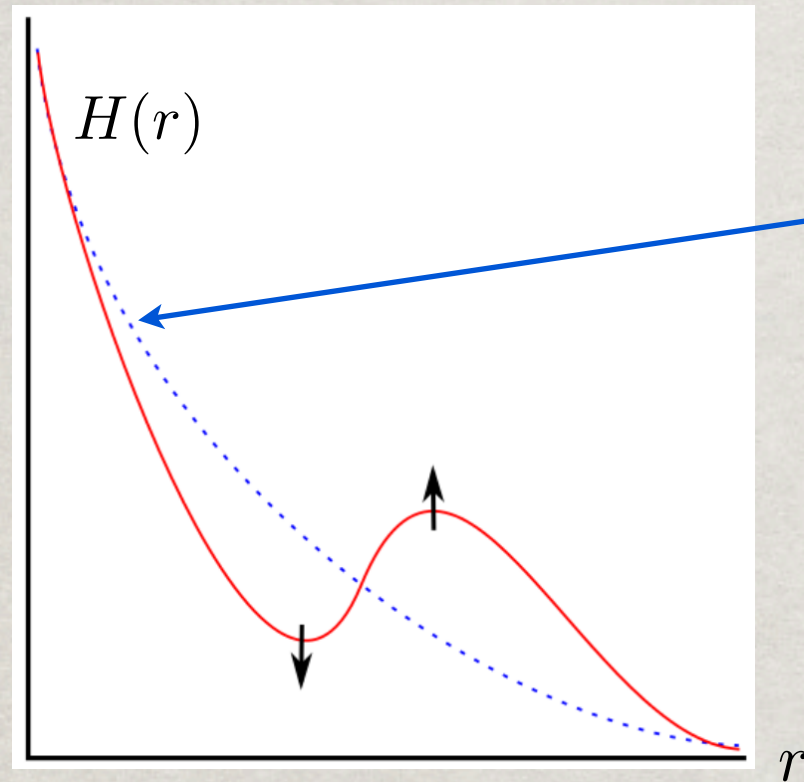
Smooth deformation fixing:

- asymptotic behavior (asymptotic flatness).
- near origin behavior (smoothness).

\Rightarrow Extrema are created in pairs.

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start: \simeq flat spacetime
Topological charge: $w = 0$

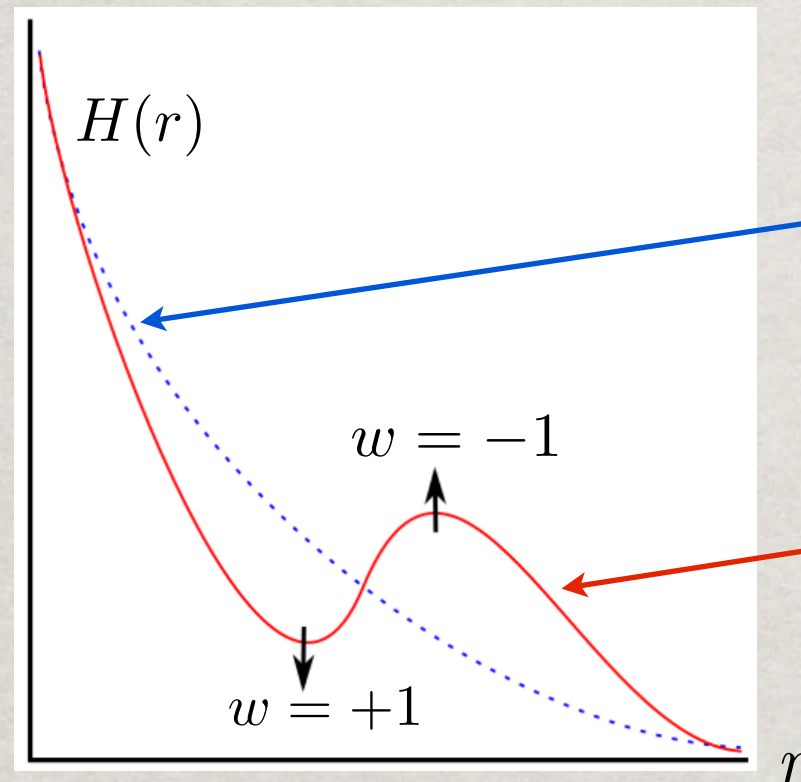
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- asymptotic behavior (asymptotic flatness).
- near origin behavior (smoothness).

\Rightarrow Extrema are created in pairs.

Light rings are created in pairs (e.g. spherical symmetry)

- In spherical symmetry \rightarrow potential $H(r)$ is 1D.



start: \simeq flat spacetime
Topological charge: $w = 0$

end: UCO
Topological charge: $w = +1 - 1 = 0$

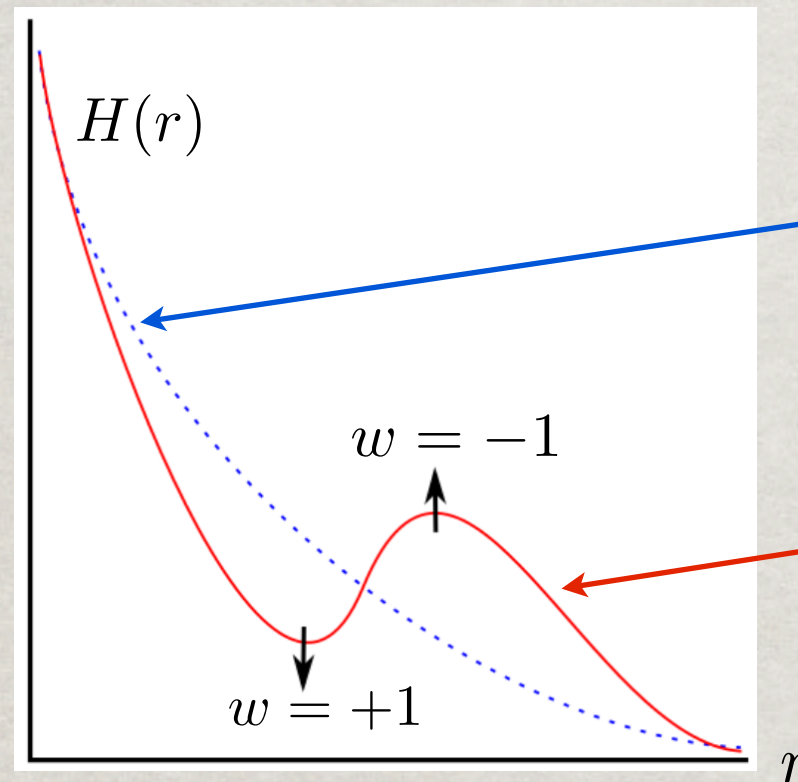
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Total topological charge
 $\sum_i w_i = \text{constant}$
if boundary conditions preserved

Light rings and Null Energy Condition

Consider Einstein's field equations:

$$G^{\mu\nu} = 8\pi T^{\mu\nu}.$$

At a Light Ring:

$$T^{\mu\nu} p_\mu p_\nu = \frac{1}{16\pi} \partial_i \partial^i U.$$

If the LR is exotic (local maximum of U):

- $\partial_i \partial^i U < 0 \implies T^{\mu\nu} p_\mu p_\nu < 0.$
- Null Energy Condition (NEC) is violated for an exotic LR!
- Enforcing NEC \implies horizonless UCO has a stable LR.

A non-linear instability ?

It has been suggested: [J. Keir, Class.Quant.Grav. 33 \(2016\) no.13, 135009](#); [Benomio, arXiv:1809.07795](#)

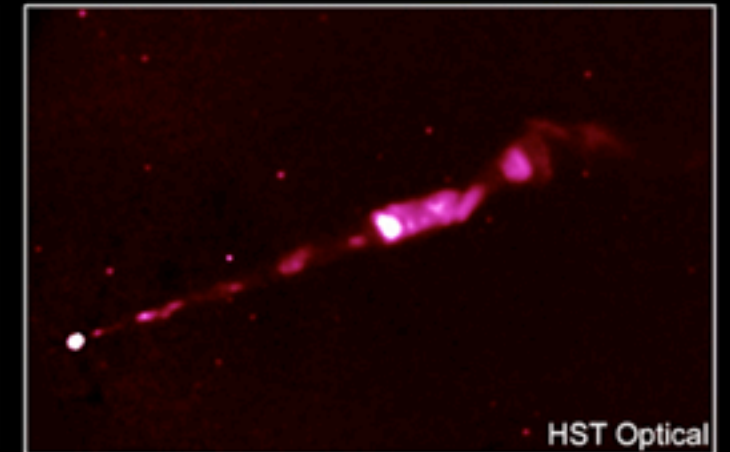
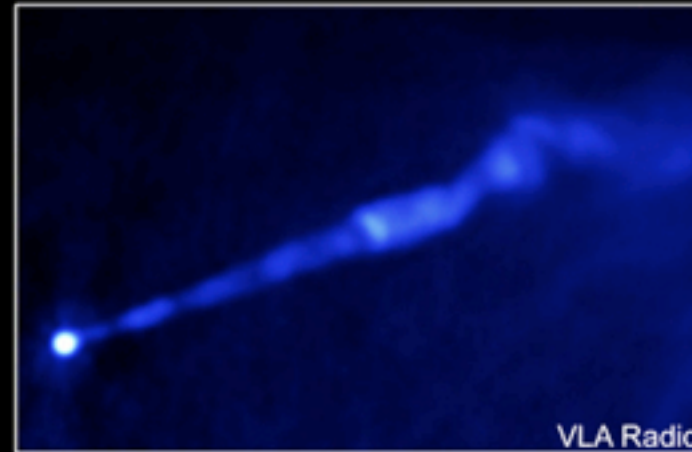
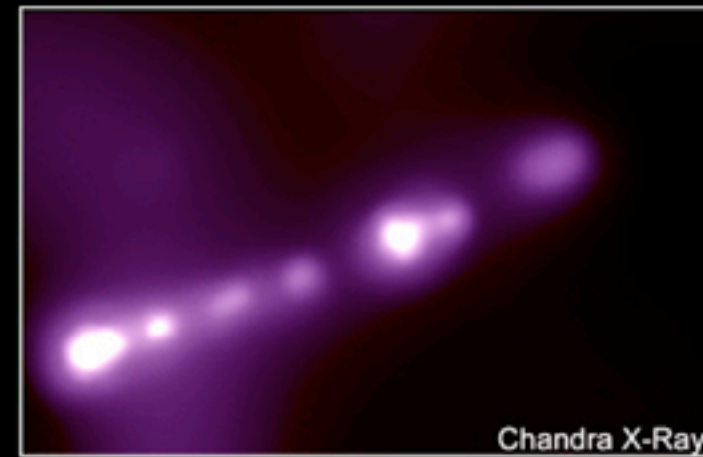
- Treating scalar linear waves as a model for nonlinear perturbations.
- Considering spherically symmetric spacetimes exhibiting stable Light Rings.
- Showing that linear waves cannot (uniformly) decay faster than logarithmically.
- Such slow decay is highly suggestive of a *nonlinear instability*.

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- Considering spherically symmetric spacetimes exhibiting stable Light Rings.
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- Such slow decay is highly suggestive of a *nonlinear instability*.

Thus, the existence of a stable light ring is a (potentially) generic obstruction for any UCO that can form from classical GR dynamics.



Plan: to discuss strong light bending

- 1) Paradigm: Kerr black holes
- 2) Non-Kerr (but reasonable) black holes
- 3) (Generic) horizonless ultracompact compact objects
- 4) Epilogue;

Light is a natural probe of spacetime geometry.



100 years ago, in this beautiful island,
weak gravitational lensing was first seen ...

so it is time strong gravitational
lensing is observed here as well...



Thank you for your attention!



Obrigado pela vossa atenção!